

Enrichment

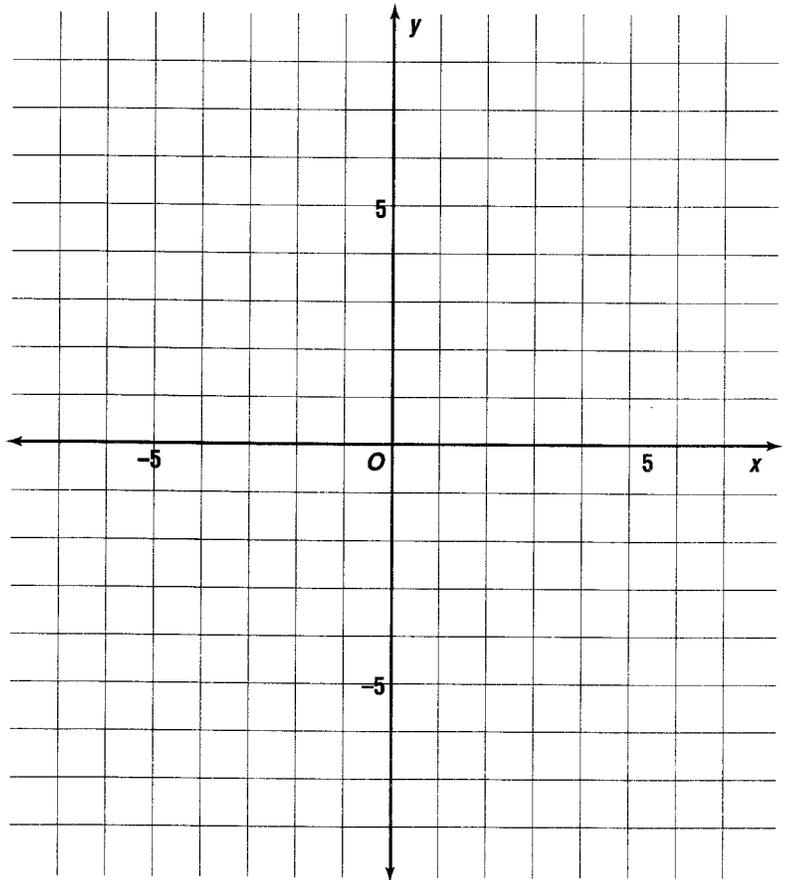
Student Edition
Pages 6-11

Coordinate Pictures

By connecting the points for two ordered pairs, you can make a line segment. Line segments can be combined to make pictures.

Graph the two ordered pairs in each exercise, and connect them with a line segment. After you draw each segment, identify the picture.

1. $(-2, 5.5)$ and $(5, 5.5)$
2. $(-3, 4)$ and $(4, 4)$
3. $(-2, 3)$ and $(3, 3)$
4. $(-2, -0.5)$ and $(3, -0.5)$
5. $(1, -1.5)$ and $(3, -1.5)$
6. $(-3, -2)$ and $(4, -2)$
7. $(-3, -3)$ and $(4, -3)$
8. $(-4, -4)$ and $(5.5, -4)$
9. $(-3, -4.5)$ and $(4, -4.5)$
10. $(-3.5, -5)$ and $(3.5, -5)$
11. $(-4, -5.5)$ and $(3, -5.5)$
12. $(-4.5, -6)$ and $(2.5, -6)$
13. $(-6.5, -6.5)$ and $(3.5, -6.5)$
14. $(-6.5, -7)$ and $(3.5, -7)$
15. $(-3, 4)$ and $(-3, -3)$
16. $(-2, -0.5)$ and $(-2, 3)$
17. $(3, -0.5)$ and $(3, 3)$
18. $(4, -3)$ and $(4, 4)$
19. $(5, -1.5)$ and $(5, 5.5)$
20. $(3.5, -6.5)$ and $(3.5, -7)$
21. $(5.5, -4)$ and $(5.5, -4.5)$
22. $(-6.5, -6.5)$ and $(-6.5, -7)$
23. $(4, -2)$ and $(5, -0.5)$
24. $(4, -3)$ and $(5, -1.5)$
25. $(-3, 4)$ and $(-2, 5.5)$
26. $(4, 4)$ and $(5, 5.5)$
27. $(-6.5, -6.5)$ and $(-4, -4)$
28. $(5.5, -4)$ and $(3.5, -6.5)$
29. $(5.5, -4.5)$ and $(3.5, -7)$
30. What does the picture show? _____

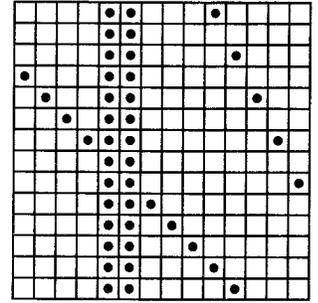


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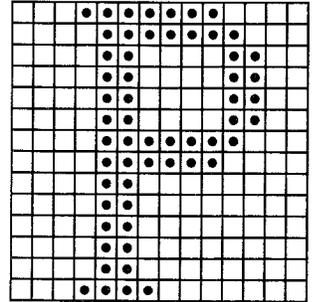
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Pages 12-18

Points and Lines on a Matrix

A **matrix** is a rectangular array of rows and columns. Points and lines on a matrix are not defined in the same way as in Euclidean geometry. A **point** on a matrix is a dot, which can be small or large. A **line** on a matrix is a path of dots that “line up.” Between two points on a line there may or may not be other points. Three examples of lines are shown at the upper right. The broad line can be thought of as a single line or as two narrow lines side by side.

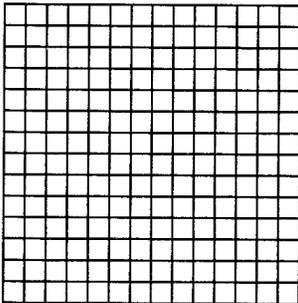


A dot-matrix printer for a computer uses dots to form characters. The dots are often called **pixels**. The matrix at the right shows how a dot-matrix printer might print the letter P.

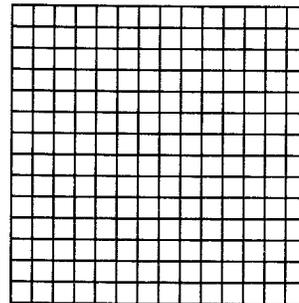


Draw points on each matrix to create the given figures.

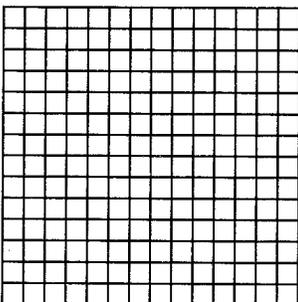
1. Draw two intersecting lines that have four points in common.



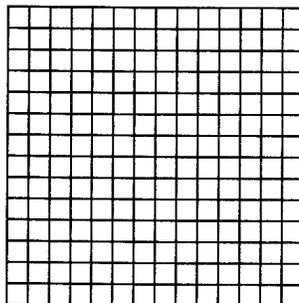
2. Draw two lines that cross but have no common points.



3. Make the number 0 (zero) so that it extends to the top and bottom sides of the matrix.



4. Make the capital letter O so that it extends to each side of the matrix.



5. Using separate grid paper, make dot designs for several other letters. Which were the easiest and which were the most difficult?

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Perimeter and Area of Irregular Shapes

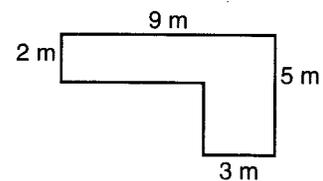
Two formulas that are used frequently in mathematics are perimeter and area of a rectangle.

Perimeter: $P = 2\ell + 2w$,

Area: $A = \ell w$, where ℓ is the length and w is the width

However, many figures are combinations of two or more rectangles creating **irregular shapes**. To find the area of an irregular shape, it helps to separate the shape into rectangles, calculate the formula for each rectangle, then find the sum of the areas.

Example: Find the area of the figure at the right.



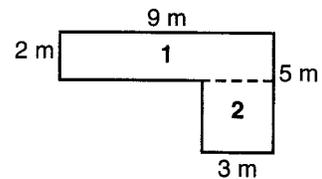
Separate the figure into two rectangles.

$$A = \ell w$$

$$A_1 = 9 \cdot 2 \qquad A_2 = 3 \cdot 3$$

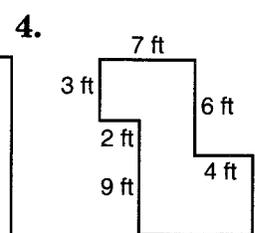
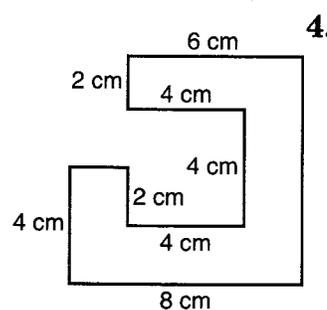
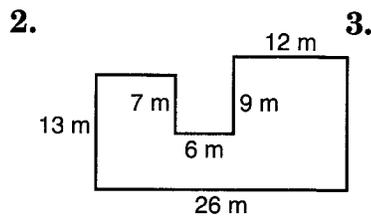
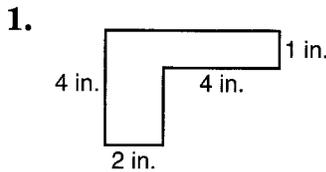
$$= 18 \qquad = 9$$

$$18 + 9 = 27$$



The area of the irregular shape is 27 m^2 .

Find the area of each irregular shape.



For questions 5-8, find the perimeter of the figures in Exercises 1-4.

5. _____ 6. _____ 7. _____ 8. _____

9. Describe the steps you used to find the perimeter in Exercise 1.

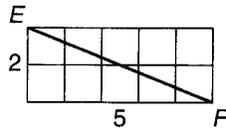
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Lengths on a Grid

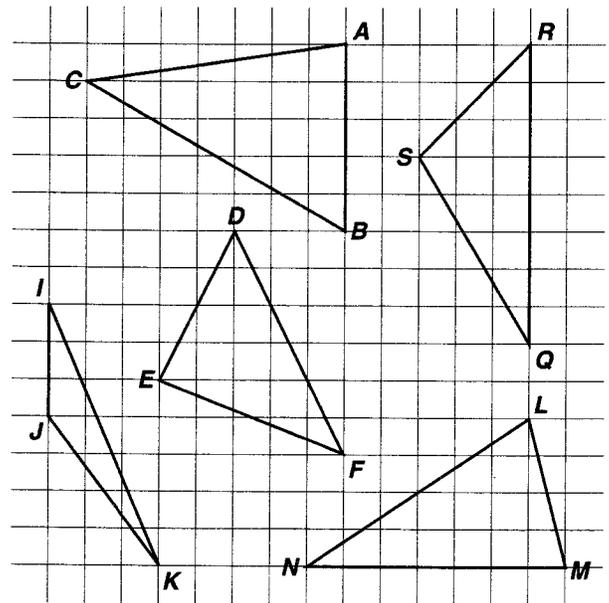
Evenly-spaced horizontal and vertical lines form a grid.

You can easily find segment lengths on a grid if the endpoints are grid-line intersections. For horizontal or vertical segments, simply count squares. For diagonal segments, use the Pythagorean Theorem (proven in Chapter 8). This theorem states that in any right triangle, if the length of the longest side (the side opposite the right angle) is c and the two shorter sides have lengths a and b , then $c^2 = a^2 + b^2$.

Example: Find the measure of \overline{EF} on the grid at the right. Locate a right triangle with \overline{EF} as its longest side.



$$EF = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.4 \text{ units}$$



Find each measure to the nearest tenth of a unit.

- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 1. \overline{IJ} | 2. \overline{MN} | 3. \overline{RS} | 4. \overline{QS} |
| 5. \overline{IK} | 6. \overline{JK} | 7. \overline{LM} | 8. \overline{LN} |

Use the grid above. Find the perimeter of each triangle to the nearest tenth of a unit.

- | | | | |
|--------------------|---------------------|---------------------|---------------------|
| 9. $\triangle ABC$ | 10. $\triangle QRS$ | 11. $\triangle DEF$ | 12. $\triangle LMN$ |
|--------------------|---------------------|---------------------|---------------------|

13. Of all the segments shown on the grid, which is longest? What is its length?

14. On the grid, 1 unit = 0.5 cm. How can the answers above be used to find the measures in centimeters?

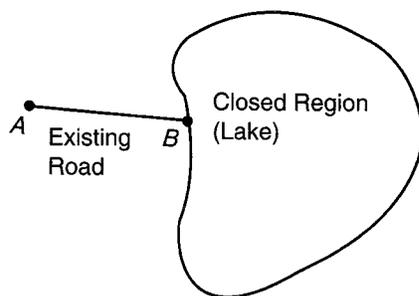
15. Use your answer from exercise 8 to calculate the length of segment \overline{LN} in centimeters. Check by measuring with a centimeter ruler.

16. Use a centimeter ruler to find the perimeter of triangle IJK to the nearest tenth of a centimeter.

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Construction Problem

The diagram below shows segment AB adjacent to a closed region. The problem requires that you construct another segment XY to the right of the closed region such that points A , B , X , and Y are collinear. You are not allowed to touch or cross the closed region with your compass or straightedge.



Follow these instructions to construct a segment XY so that it is collinear with segment AB .

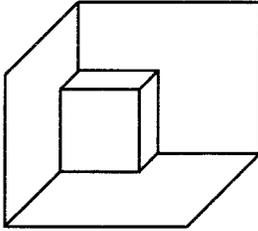
1. Construct the perpendicular bisector of \overline{AB} . Label the midpoint as point C , and the line as m .
2. Mark two points P and Q on line m that lie well above the closed region. Construct the perpendicular bisector n of \overline{PQ} . Label the intersection of lines m and n as point D .
3. Mark points R and S on line n that lie well to the right of the closed region. Construct the perpendicular bisector ℓ of \overline{RS} . Label the intersection of lines n and ℓ as point E .
4. Mark point X on line ℓ so that X is below line n and so that \overline{EX} is congruent to \overline{DC} .
5. Mark points T and V on line ℓ and on opposite sides of X , so that \overline{XT} and \overline{XV} are congruent. Construct the perpendicular bisector ℓ' of \overline{TV} . Call the point where the line ℓ' hits the boundary of the closed region point Y . \overline{XY} corresponds to the new road.

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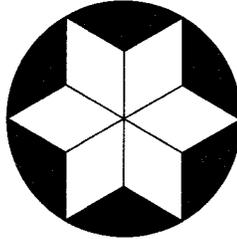
Optical Illusions

Study each figure. Determine how many interpretations might be seen. Describe each interpretation.

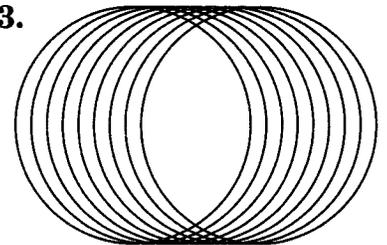
1.



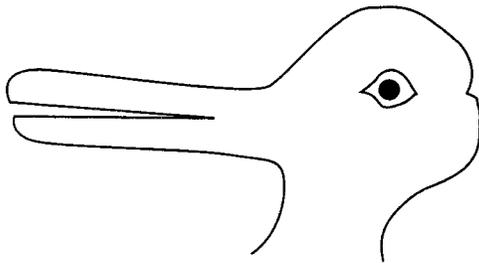
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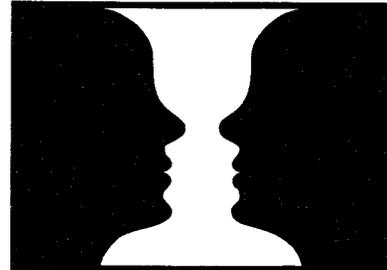
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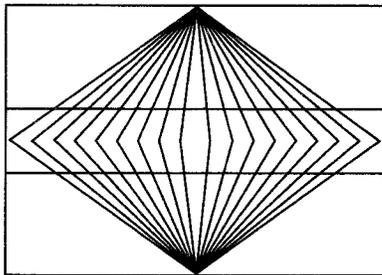
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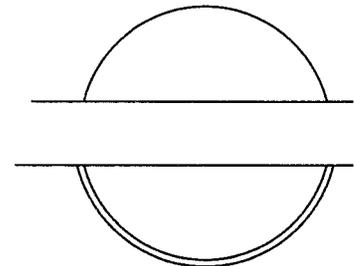
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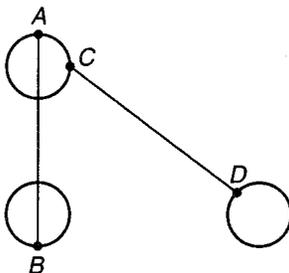
6. Are the two horizontal segments straight or curved?



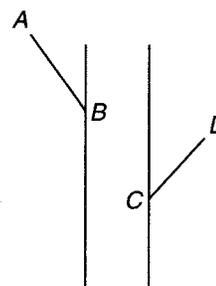
7. Does the inner arc or the outer arc in the lower part of the figure go with the upper arcs to form part of a circle?



8. Is the segment from A to B as long as the segment from C to D ?



9. If extended, will the segment from A to B cross the segment from C to D at C ?



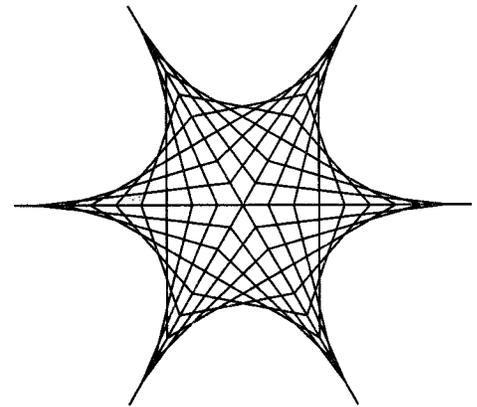
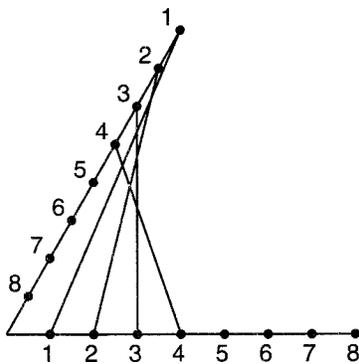
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Curve Stitching

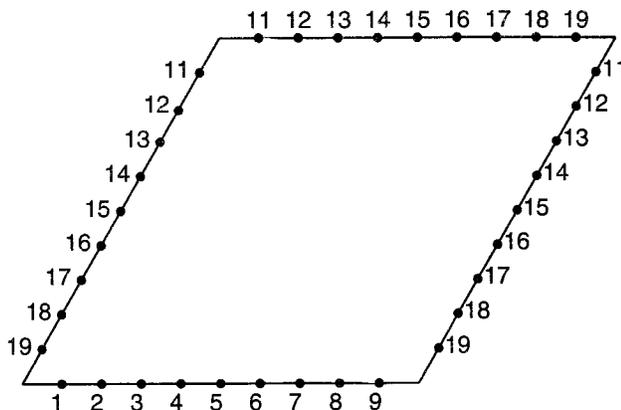
The star design at the right was created by a method known as **curve stitching**. Although the design appears to contain curves, it is made up entirely of line segments.

To begin the star design, draw a 60° angle. Mark eight equally-spaced points on each ray, and number the points as shown below. Then connect pairs of points that have the same number.



To make a complete star, make the same design in six 60° angles that have a common central vertex.

1. Complete the section of the star design above by connecting pairs of points that have the same number.
2. Complete the following design.



3. Create your own design. You may use several angles, and the angles may overlap.

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Counterexamples

When you make a conclusion after examining several specific cases, you have used **inductive reasoning**. However, you must be cautious when using this form of reasoning. By finding only one **counterexample**, you disprove the conclusion.

Example: Is the statement $\frac{1}{x} \leq 1$ true when you replace x with 1, 2, and 3? Is the statement true for all reals? If possible, find a counterexample.
 $\frac{1}{1} = 1$, $\frac{1}{2} < 1$, and $\frac{1}{3} < 1$. But when $x = \frac{1}{2}$, then $\frac{1}{x} = 2$. This counterexample shows that the statement is not always true.

Answer each question.

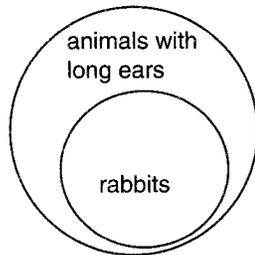
1. The coldest day of the year in Chicago occurred in January for five straight years. Is it safe to conclude that the coldest day in Chicago is always in January?
2. Suppose John misses the school bus four Tuesdays in a row. Can you safely conclude that John misses the school bus every Tuesday?
3. Is the equation $\sqrt{k^2} = k$ true when you replace k with 1, 2, and 3? Is the equation true for all integers? If possible, find a counterexample.
4. Is the statement $2x = x + x$ true when you replace x with $\frac{1}{2}$, 4, and 0.7? Is the statement true for all real numbers? If possible, find a counterexample.
5. Suppose you draw four points A , B , C , and D and then draw \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} . Does this procedure give a quadrilateral always or only sometimes? Explain your answers with figures.
6. Suppose you draw a circle, mark three points on it, and connect them. Will the angles of the triangle be acute? Explain your answers with figures.

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Venn Diagrams

A type of drawing called a **Venn diagram** can be useful in explaining conditional statements. A Venn diagram uses circles to represent sets of objects.

Consider the statement "All rabbits have long ears." To make a Venn diagram for this statement, a large circle is drawn to represent all animals with long ears. Then a smaller circle is drawn inside the first to represent all rabbits. The Venn diagram shows that every rabbit is included in the group of long-eared animals.



The set of rabbits is called a **subset** of the set of long-eared animals.

The Venn diagram can also explain how to write the statement, "All rabbits have long ears," in if-then form. Every rabbit is in the group of long-eared animals, so if an animal is a rabbit, then it has long ears.

For each statement, draw a Venn diagram. Then write the sentence in if-then form.

1. Every dog has long hair.

2. All rational numbers are real.

3. People who live in Iowa like corn.

4. Staff members are allowed in the faculty lounge.

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Valid and Faulty Arguments

Consider the statements at the right.
 What conclusions can you make?

- (1) Boots is a cat.
- (2) Boots is purring.
- (3) A cat purrs if it is happy.

From statements 1 and 3, it is correct to conclude that Boots purrs if it is happy. However, it is faulty to conclude from only statements 2 and 3 that Boots is happy. The if-then form of statement 3 is *If a cat is happy, then it purrs.*

Advertisers often use faulty logic in subtle ways to help sell their products. By studying the arguments, you can decide whether the argument is valid or faulty.

Decide if each argument is valid or faulty.

1. (1) If you buy Tuff Cote luggage, it will survive airline travel.
 (2) Justin buys Tuff Cote luggage.
 Conclusion: Justin's luggage will survive airline travel.
2. (1) If you buy Tuff Cote luggage, it will survive airline travel.
 (2) Justin's luggage survived airline travel.
 Conclusion: Justin has Tuff Cote luggage.
3. (1) If you use Clear Line long distance service, you will have clear reception.
 (2) Anna has clear long distance reception.
 Conclusion: Anna uses Clear Line long distance service.
4. (1) If you read the book *Beautiful Braids*, you will be able to make beautiful braids easily.
 (2) Nancy read the book *Beautiful Braids*.
 Conclusion: Nancy can make beautiful braids easily.
5. (1) If you buy a word processor, you will be able to write letters faster.
 (2) Tania bought a word processor.
 Conclusion: Tania will be able to write letters faster.
6. (1) Great swimmers wear AquaLine swimwear.
 (2) Gina wears AquaLine swimwear.
 Conclusion: Gina is a great swimmer.
7. Write an example of faulty logic that you have seen in an advertisement.

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Symmetric, Reflexive, and Transitive Properties

Equality has three important properties.

Symmetric $a = a$

Reflexive If $a = b$, then $b = a$.

Transitive If $a = b$ and $b = c$, then $a = c$.

Other relations have some of the same properties. Consider the relation "is next to" for objects labeled X , Y , and Z . Which of the properties listed above are true for this relation?

X is next to X . *False*

If X is next to Y , then Y is next to X . *True*

If X is next to Y and Y is next to Z , then X is next to Z . *False*

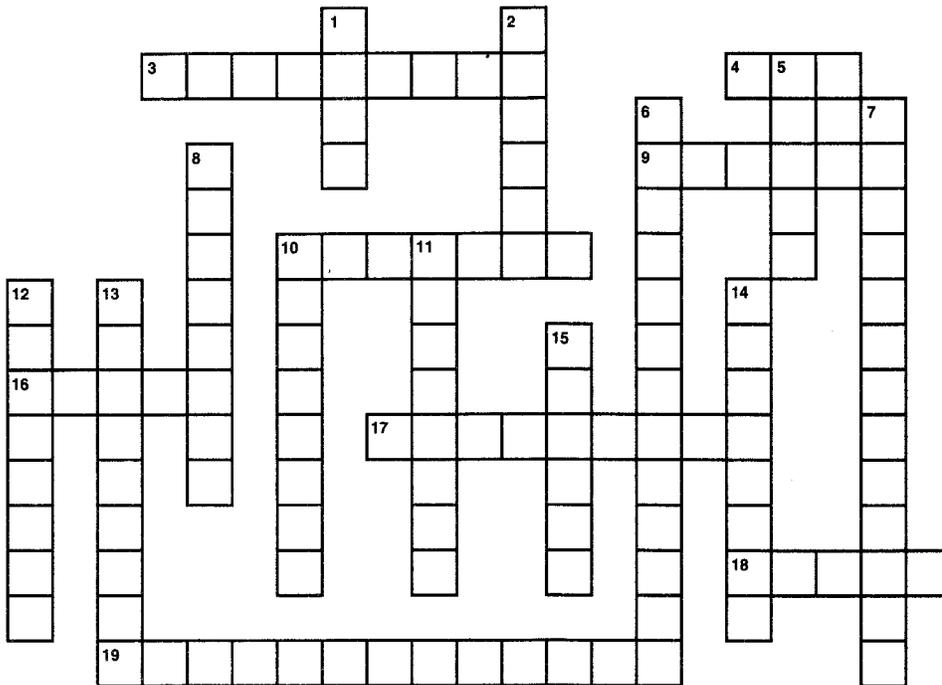
Only the symmetric property is true for the relation "is next to."

For each relation, state which properties (symmetric, reflexive, transitive) are true.

- | | |
|---------------------------|------------------------------|
| 1. is the same size as | 2. is a family descendant of |
| 3. is in the same room as | 4. is the identical twin of |
| 5. is warmer than | 6. is on the same line as |
| 7. is a sister of | 8. is the same weight as |
9. Find two other examples of relations, and tell which properties are true for each relation.

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Geometry Crossword Puzzle



ACROSS

3. Points on the same line are _____.
4. A point on a line and all points of the line to one side of it.
9. An angle whose measure is greater than 90.
10. Two endpoints and all points between them.
16. A flat figure with no thickness that extends indefinitely in all directions.
17. Segments of equal length are _____ segments.
18. Two noncollinear rays with a common endpoint.
19. If $m\angle A + m\angle B = 180$, then $\angle A$ and $\angle B$ are _____ angles.

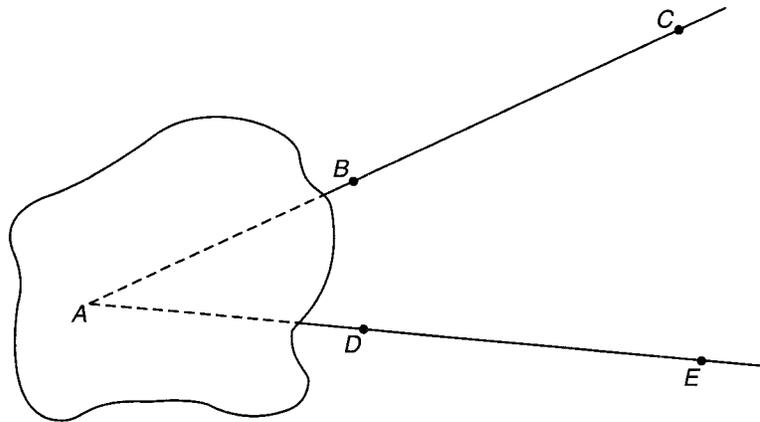
DOWN

1. The set of all points collinear to two points is a _____.
2. The point where the x - and y -axis meet.
5. An angle whose measure is less than 90.
6. If $m\angle A + m\angle D = 90$, then $\angle A$ and $\angle D$ are _____ angles.
7. Lines that meet at a 90° angle are _____.
8. Two angles with a common side but no common interior points are _____.
10. An "angle" formed by opposite rays is a _____ angle.
11. The middle point of a line segment.
12. Points that lie in the same plane are _____.
13. The four parts of a coordinate plane.
14. Two nonadjacent angles formed by two intersecting lines are _____ angles.
15. In angle ABC , point B is the _____.

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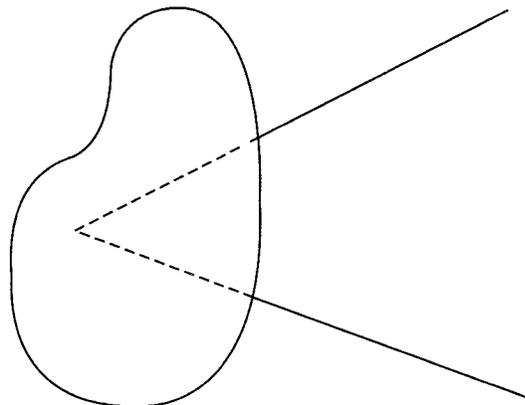
Bisecting a Hidden Angle

The vertex of $\angle BAD$ at the right is hidden in a region. Within the region, you are not allowed to use a compass. Can you bisect the angle?



Follow these instructions to bisect $\angle BAD$.

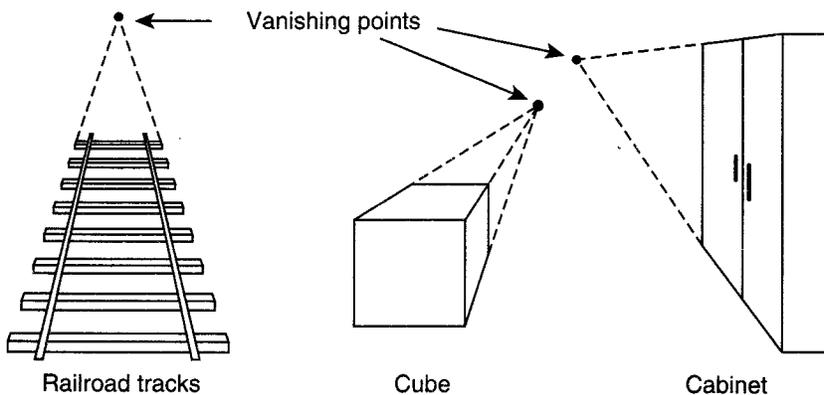
1. Use a straightedge to draw lines CE and BD .
2. Construct the bisectors of $\angle DEC$ and $\angle BCE$.
3. Label the intersection of the two bisectors as point P .
4. Construct the bisectors of $\angle BDE$ and $\angle DBC$.
5. Label the intersection of the two previous bisectors as point Q .
6. Use a straightedge to draw line PQ , which bisects the hidden angle.
7. Another hidden angle is shown at right. Construct the bisector using the method above, or devise your own method.



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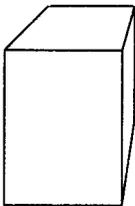
Perspective Drawings

To draw three-dimensional objects, artists make **perspective drawings** such as the ones shown below. To indicate depth in a perspective drawing, some parallel lines are drawn as converging lines. The dotted lines in the figures below each extend to a **vanishing point**, or spot where parallel lines appear to meet.

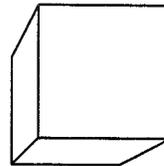


Draw lines to locate the vanishing point in each drawing of a box.

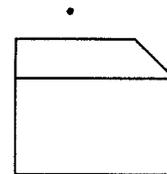
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2.

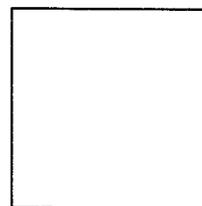
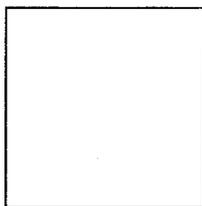


3.



4. The fronts of two cubes are shown below. Using point P as the vanishing point for both cubes, complete the perspective drawings of the cubes.

P



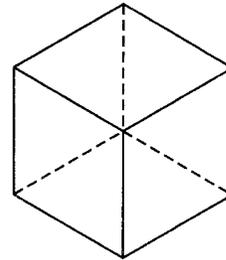
5. Find an example of a perspective drawing in a newspaper or magazine. Trace the drawing and locate a vanishing point.

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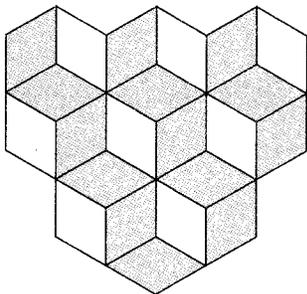
More Optical Illusions

In drawings, diagonal lines may create the illusion of depth. For example, the figure at the right can be thought of as picturing a flat figure or a cube. The optical illusions on this page involve depth perception.

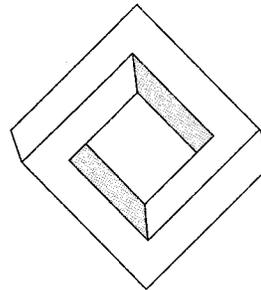


Answer each question.

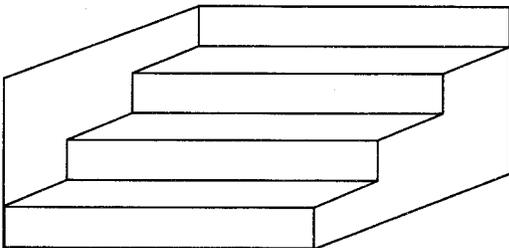
1. How many cubes do you see in the drawing?



2. Can this figure show an actual object?

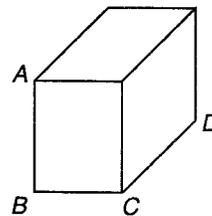


3.



Does the drawing show a view from the top or the bottom of the stairs?

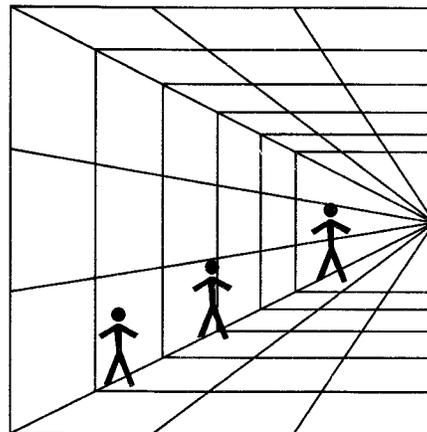
4.



Which line segment is longer, \overline{AB} or \overline{CD} ? Measure to check your answer.

5. Which person in the drawing at the right appears to be tallest? Measure to check your answer.

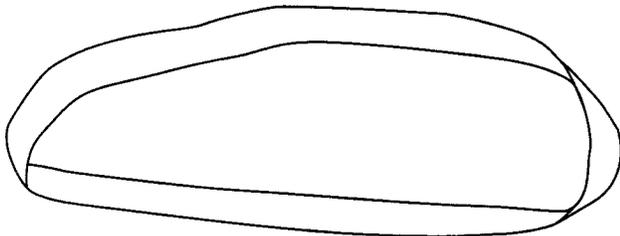
6. Draw two more objects the same size on the figure at the right. Does one appear larger than the other?



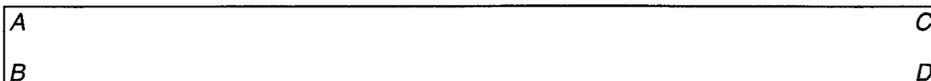
Enrichment

The Möbius Strip

A Möbius strip is a special surface with only one side. It was discovered by August Ferdinand Möbius, a German astronomer and mathematician.



1. To make a Möbius strip, cut a strip of paper about 16 inches long and 1 inch wide. Mark the ends with the letters *A*, *B*, *C*, and *D* as shown below.



Twist the paper once, connecting *A* to *D* and *B* to *C*. Tape the ends together on both sides.

2. Use a crayon or pencil to shade one side of the paper. Shade around the strip until you get back to where you started. What happens?
3. What do you think will happen if you cut the Möbius strip down the middle? Try it.
4. Make another Möbius strip. Starting a third of the way in from one edge, cut around the strip, staying always the same distance in from the edge. What happens?
5. Start with another long strip of paper. Twist the paper twice and connect the ends. What happens when you cut down the center of this strip?
6. Start with another long strip of paper. Twist the paper three times and connect the ends. What happens when you cut down the center of this strip?

Enrichment

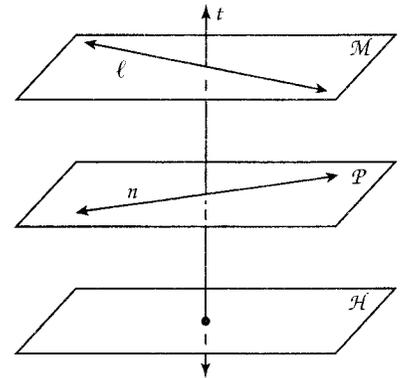
Parallelism in Space

In space geometry, the concept of parallelism must be extended to include two planes and a line and a plane.

Definition: Two planes are parallel if and only if they do not intersect.

Definition: A line and a plane are parallel if and only if they do not intersect.

Thus, in space, two lines can be intersecting, parallel, or skew while two planes or a line and a plane can only be intersecting or parallel. In the figure at the right, $t \perp \mathcal{M}$, $t \perp \mathcal{P}$, $\mathcal{P} \parallel \mathcal{H}$, and ℓ and n are skew.



The following five statements are theorems about parallel planes.

Theorem: Two planes perpendicular to the same line are parallel.

Theorem: Two planes parallel to the same plane are parallel.

Theorem: A line perpendicular to one of two parallel planes is perpendicular to the other.

Theorem: A plane perpendicular to one of two parallel planes is perpendicular to the other.

Theorem: If two parallel planes each intersect a third plane, then the two lines of intersection are parallel.

Use the figure given above for Exercises 1 to 10. State yes or no to tell whether the statement is true.

- | | | | |
|--|------------------------------|--|---------------------------------|
| 1. $\mathcal{M} \parallel \mathcal{P}$ | 2. $\ell \parallel n$ | 3. $\mathcal{M} \parallel \mathcal{H}$ | 4. $\ell \parallel \mathcal{P}$ |
| 5. $\ell \perp t$ | 6. $n \parallel \mathcal{H}$ | 7. $\ell \perp \mathcal{P}$ | 8. $t \parallel \mathcal{H}$ |
| 9. $\mathcal{M} \perp t$ | 10. $t \perp \mathcal{H}$ | | |

For Exercises 11 to 14, make a small sketch at the bottom of the page to show that each statement is false.

11. If two lines are parallel to the same plane, then the lines are parallel.
12. If two planes are parallel, then any line in one plane is parallel to any line in the other plane.
13. If two lines are parallel, then any plane containing one of the lines is parallel to any plane containing the other line.
14. If two lines are parallel, then any plane containing one of the lines is parallel to the other line.

11.

12.

13.

14.

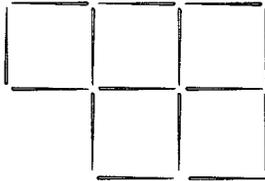
Enrichment

Toothpick Puzzles

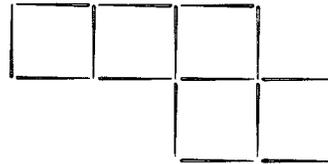
For these puzzles, you will need toothpicks or short straws of equal length.

1. Make four triangles with six toothpicks.
2. Make six triangles and a hexagon with six toothpicks.

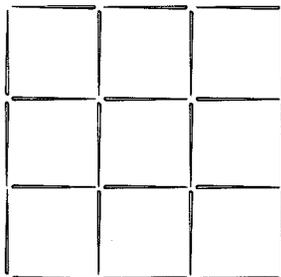
3. Use 15 toothpicks to make this arrangement. Then take away three toothpicks to leave three squares.



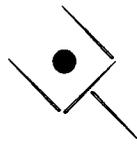
4. Use 16 toothpicks to make this arrangement. Then move two toothpicks to make four squares.



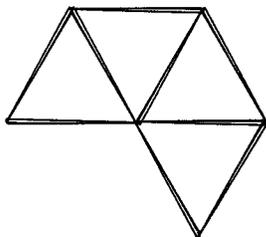
5. Use 24 toothpicks to make this arrangement. Then take away six toothpicks to leave three squares.



6. Use four toothpicks and a marble (or other small object) to make this fork arrangement. Move two toothpicks to make another fork, but with the marble no longer between the prongs of the fork.



7. Use nine toothpicks to make this arrangement. Then move two toothpicks to make a figure that creates the illusion of a cube viewed from one corner.



8. Make your own toothpick puzzle.

Enrichment

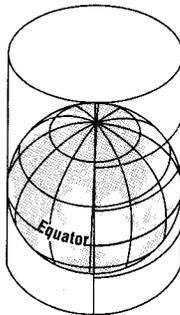
Student Edition
Pages 163-169

Map Projections

A **map projection** is a medium for transferring the surface of Earth to a flat surface map. Projection maps often distort the shapes of countries and bodies of water. You cannot measure distance accurately on such maps. Countries of equal area often do not appear to be the same size. The lines of latitude and longitude become the guidelines for your orientation in reading the map.

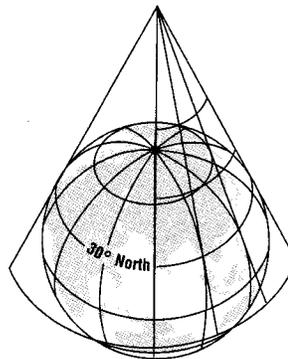
Three of the basic types of map projections are shown here. Each type of projection can be visualized by imagining a piece of film in a certain shape being exposed to light coming from within the globe.

cylindrical projection



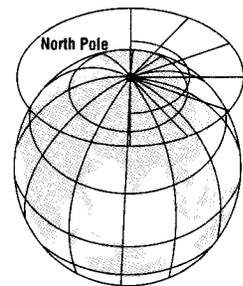
The film is a rectangle wrapped around the globe. It touches the globe at the equator.

conic projection



The film is a cone sitting on top of the globe. It touches the globe at one of the latitude lines.

azimuthal projection



The film is a circular disk that touches the globe at the pole.

Sketch an example of what the film would look like for each type of map projection.

1. cylindrical projection

2. conic projection

3. azimuthal projection

Study each type of projection. At what point(s) would the distortion to the actual shapes of the countries be the least?

4. cylindrical projection

5. conic projection

6. azimuthal

Enrichment

Reading Mathematics

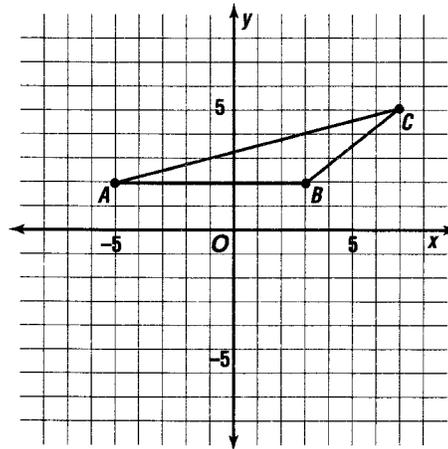
When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example: Consider three points, A , B , and C on a coordinate grid. The y -coordinates of A and B are the same. The x -coordinate of B is greater than the x -coordinate of A . Both coordinates of C are greater than the corresponding coordinates of B . Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description.

Side AB must be a horizontal segment because the y -coordinates are the same. Point C must be located to the right and up from point B .

From the diagram you can see that triangle ABC must be obtuse.



Answer each question. Draw a simple triangle on the grid above to help you.

- Consider three points, R , S , and T on a coordinate grid. The x -coordinates of R and S are the same. The y -coordinate of T is between the y -coordinates of R and S . The x -coordinate of T is less than the x -coordinate of R . Is angle R of triangle RST acute, right, or obtuse?
- Consider three noncollinear points, J , K , and L on a coordinate grid. The y -coordinates of J and K are the same. The x -coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse?
- Consider three noncollinear points, D , E , and F on a coordinate grid. The x -coordinates of D and E are opposites. The y -coordinates of D and E are the same. The x -coordinate of F is 0. What kind of triangle must $\triangle DEF$ be: scalene, isosceles, or equilateral?
- Consider three points, G , H , and I on a coordinate grid. Points G and H are on the positive y -axis, and the y -coordinate of G is twice the y -coordinate of H . Point I is on the positive x -axis, and the x -coordinate of I is greater than the y -coordinate of G . Is triangle GHI scalene, isosceles, or equilateral?

Enrichment

Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example: In triangle ABC , $m\angle A$, is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m\angle B$.

$$m\angle A + m\angle B + m\angle C = 180$$

$$2x + x + (x + 8) = 180$$

$$4x + 8 = 180$$

$$4x = 172$$

$$x = 43$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

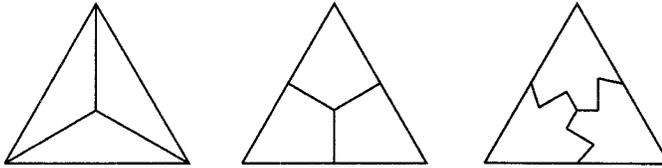
Solve each problem.

- In triangle DEF , $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?
- In triangle RST , $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?
- In triangle JKL , $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?
- In triangle XYZ , $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?
- In triangle GHI , $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?
- In triangle MNO , $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?
- In triangle STU , $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?
- In triangle PQR , $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?
- Write your own problems about measures of triangles.

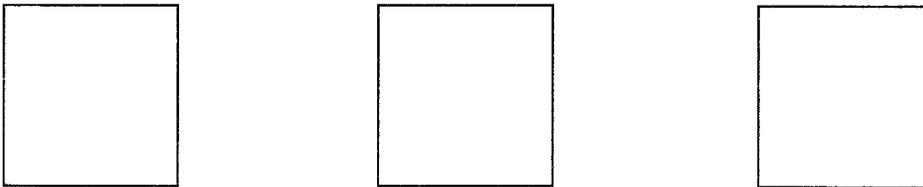
Enrichment

Congruent Parts of Regular Polygonal Regions

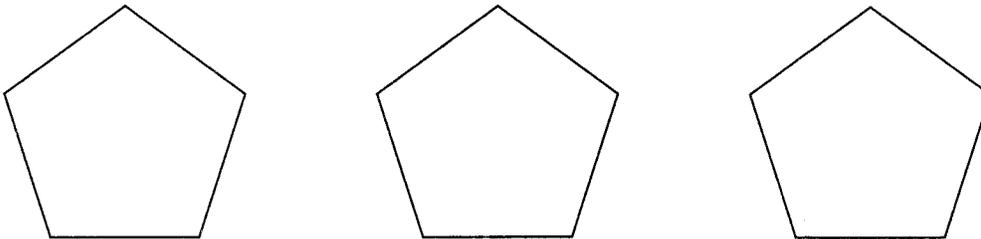
Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.



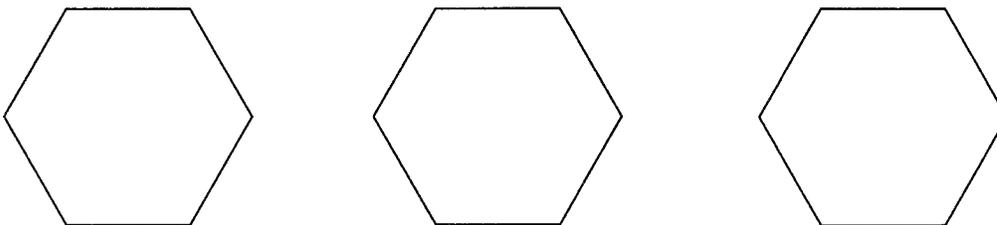
1. Divide each square into four congruent parts. Use three different ways.



2. Divide each pentagon into five congruent parts. Use three different ways.



3. Divide each hexagon into six congruent parts. Use three different ways.



4. What hints might you give another student who is trying to divide figures like those into congruent parts?

Enrichment

Student Edition
Pages 206–213

Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider $\triangle ABD$ and $\triangle CDB$ whose vertices have coordinates $A(0, 0)$, $B(2, 5)$, and $D(7, 0)$. Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that $\triangle ABD \cong \triangle CDB$. You may wish to make a sketch to help get you started.

2. Consider $\triangle PQR$ and $\triangle KLM$ whose vertices are the following points.

$P(1, 2)$

$Q(3, 6)$

$R(6, 5)$

$K(-2, 1)$

$L(-6, 3)$

$M(-5, 6)$

Briefly describe how you can show that $\triangle PQR \cong \triangle KLM$.

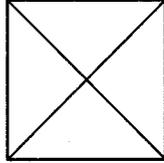
3. If you know the coordinates of all the vertices of two triangles, is it *always* possible to tell whether the triangles are congruent? Explain.

How Many Triangles?

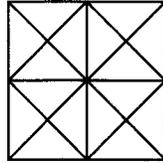
Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

How many triangles in each figure?

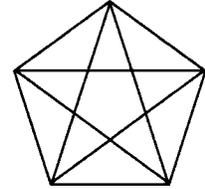
1.



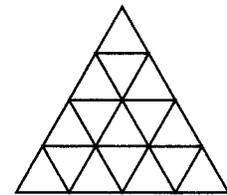
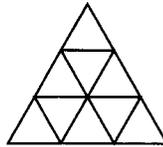
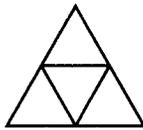
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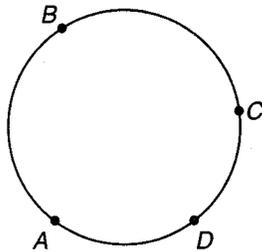


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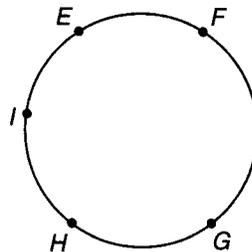


How many triangles can you form by joining points on each circle? List the vertices of each triangle.

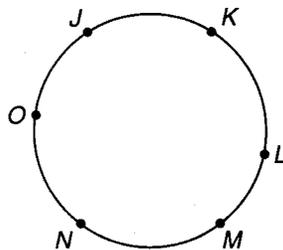
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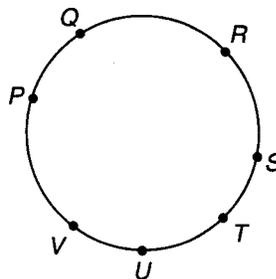
8.



9.



10.



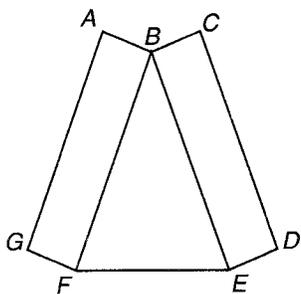
Enrichment

Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

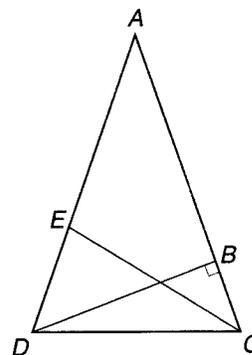
1. Given: $BE = BF$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.

Find $m\angle ABC$.



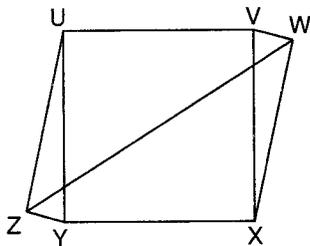
2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$,
 $m\angle DAC = 44$ and \overline{CE} bisects $\angle ACD$.

Find $m\angle DEC$.



3. Given: $m\angle UZY = 90$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).

Find $m\angle WZY$.



4. Given: $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$,
 $\triangle JNM \cong \triangle KLM$.

Find $m\angle JKM$.

