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Study Guide

Student Edition
Pages 238–244

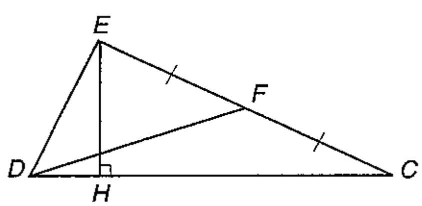
Special Segments in Triangles

Four special types of segments are associated with triangles.

- A **median** is a segment that connects a vertex of a triangle to the midpoint of the opposite side.
- An **altitude** is a segment that has one endpoint at a vertex of a triangle and the other endpoint on the line containing the opposite side so that the altitude is perpendicular to that line.
- An **angle bisector** of a triangle is a segment that bisects an angle of the triangle and has one endpoint at the vertex of that angle and the other endpoint on the side opposite that vertex.
- A **perpendicular bisector** is a segment or line that passes through the midpoint of a side and is perpendicular to that side.

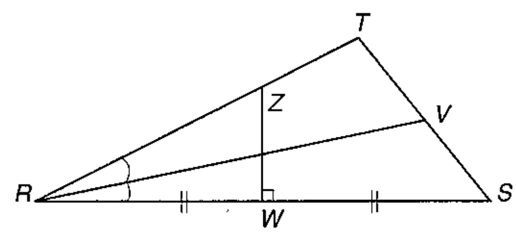
Examples:

1



$\overline{DF}$ is a median of $\triangle DEC$.
$\overline{EH}$ is an altitude of $\triangle DEC$.

2



$\overline{RV}$ is an angle bisector of $\triangle RST$.
$\overline{WZ}$ is a perpendicular bisector of side $\overline{RS}$.

Draw and label a figure to illustrate each situation.

1. $\overline{OQ}$ is a median and an altitude of $\triangle POM$.

2. $\overline{KT}$ is an altitude of $\triangle KLM$, and L is between T and M.

3. $\overline{HS}$ is an angle bisector of $\triangle GHI$, and S is between G and I.

4. $\triangle NRW$ is a right triangle with right angle at N. $\overline{NX}$ is a median of $\triangle NRW$. $\overline{YX}$ is a perpendicular bisector of $\overline{WR}$.

5. $\triangle TRE$ has vertices $T(3, 6)$, $R(-3, 10$, and $E(-9, 4)$. Find the coordinates of point M if $\overline{TM}$ is a median of $\triangle TRE$.

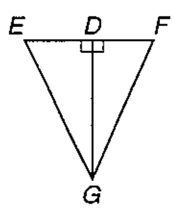
Right Triangles

Two right triangles are congruent if one of the following conditions exist.

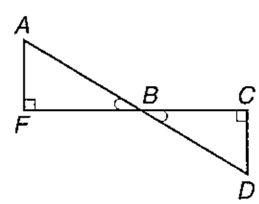
Theorem 5-5 LL	If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.
Theorem 5-6 HA	If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.
Theorem 5-7 LA	If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.
Postulate 5-1 HL	If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

State the additional information needed to prove each pair of triangles congruent by the given theorem or postulate.

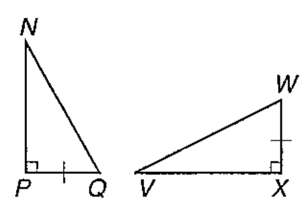
1. HL



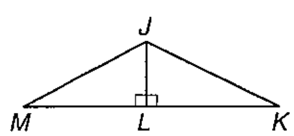
2. HA



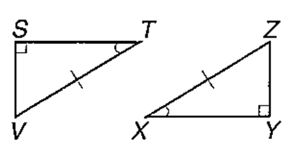
3. LL



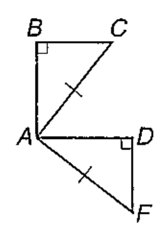
4. LA



5. HA



6. LA



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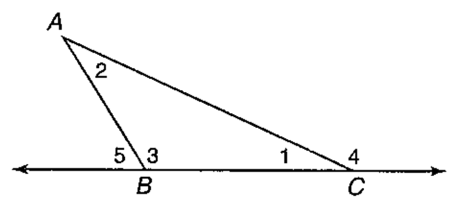
Indirect Proof and Inequalities

A type of proof called **indirect proof** is sometimes used in geometry. In an indirect proof you assume that the conclusion is false and work backward to show that this assumption leads to a contradiction of the original hypothesis or some other known fact, such as a postulate, theorem, or corollary.

The following theorem can be proved by an indirect proof. (See page 253 in your book.)

Exterior Angle Inequality Theorem	If an angle is an exterior angle of a triangle, then its measure is greater than the measure either of its corresponding remote interior angles.

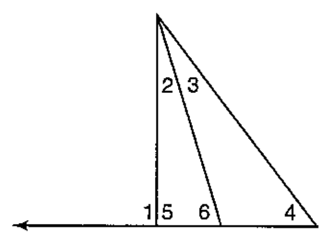
Example: Use the figure at the right to complete the statement with either < or >.



$m\angle 1$? $m\angle 5$

Since $\angle 5$ is an exterior angle of $\triangle ABC$ and $\angle 1$ and $\angle 2$ are the corresponding remote interior angles, you know that $m\angle 1 < m\angle 5$ by the Exterior Angle Inequality Theorem.

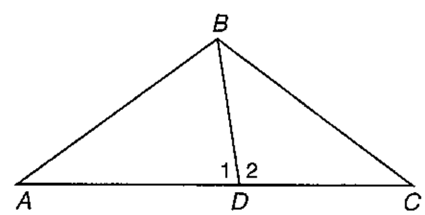
Use the figure at the right to complete each statement with < or >.



1. $m\angle 1$ ______ $m\angle 6$ **2.** $m\angle 2$ ______ $m\angle 1$

3. $m\angle 6$ ______ $m\angle 3$ **4.** $m\angle 4$ ______ $m\angle 6$

5. Use the problem-solving strategy of working backward to complete the indirect proof in paragraph form.
Given: $m\angle 1 \neq m\angle 2$
Prove: BD is not an altitude of $\triangle ABC$.



Proof:

a. Assume that ______________________________.

b. Then $\overline{BD} \perp \overline{AC}$ by ______________________________.

c. Since ______________________________, $\angle 1$ and $\angle 2$ are right angles.

d. Since all right angles are congruent, ______________________________.

e. Since $\angle 1 \cong \angle 2$, $m\angle 1 =$ ______________________________.

f. But it is given that ______________________________.

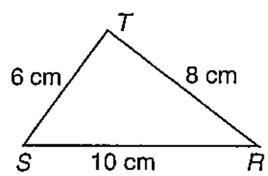
g. So our assumption is incorrect. Therefore, ______________________________.

Inequalities for Sides and Angles of a Triangle

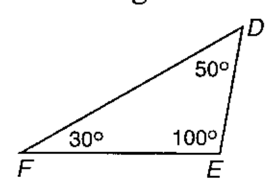
Two theorems are very useful for determining relationships between sides and angles of triangles.

- If one side of a triangle is longer than another side, then the angle opposite the longer side is greater than the angle opposite the shorter side.
- If one angle of a triangle is greater than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

Examples: ***1*** List the angles in order from least to greatest measure.

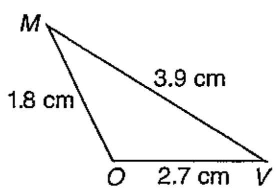


2 List the sides in order from shortest to longest.

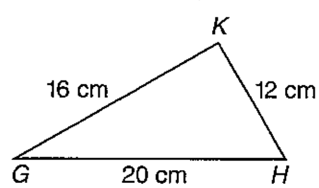


For each triangle, list the angles in order from least to greatest measure.

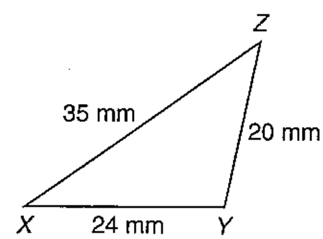
1.



2.

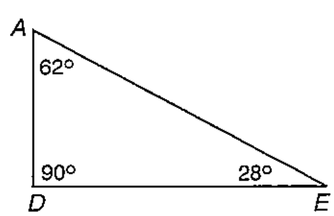


3.

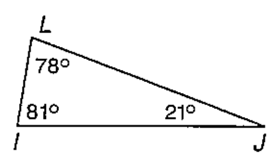


For each triangle, list the sides in order from shortest to longest.

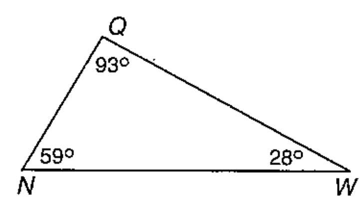
4.



5.



6.



List the sides of △ABC in order from longest to shortest if the angles of △ABC have the indicated measures.

7. $m\angle A = 5x + 2$, $m\angle B = 6x - 10$, $m\angle C = x + 20$

8. $m\angle A = 10x$, $m\angle B = 5x - 17$, $m\angle C = 7x - 1$

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The Triangle Inequality

If you take three straws that are 8 inches, 4 inches, and 3 inches in length, can you use these three straws to form a triangle? Without actually trying it, you might think it is possible to form a triangle with the straws. If you try it, however, you will notice that the two smaller straws are too short. This example illustrates the following theorem.

Triangle Inequality Theorem	The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example: If the lengths of two sides of a triangle are 7 centimeters and 11 centimeters, between what two numbers must the measure of the third side fall?

Let x = the length of the third side.

By the Triangle Inequality Theorem, each of these inequalities must be true.

$x + 7 > 11$ | $x + 11 > 7$ | $11 + 7 > x$

$x > 4$ | $x > -4$ | $18 > x$

Therefore, x must be between 4 centimeters and 18 centimeters.

Determine whether it is possible to draw a triangle with sides of the given measures. Write <u>yes</u> or <u>no</u>.

1. 15, 12, 9 **2.** 23, 16, 7 **3.** 20, 10, 9

4. 8.5, 6.5, 13.5 **5.** 47, 28, 70 **6.** 28, 41, 13

The measures of two sides of a triangle are given. Between what two numbers must the measure of the third side fall?

7. 9 and 15 **8.** 11 and 20 **9.** 23 and 14

10. Suppose you have three different positive numbers arranged in order from greatest to least. Which sum is it most crucial to test to see if the numbers could be the lengths of the sides of a triangle?

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5–6

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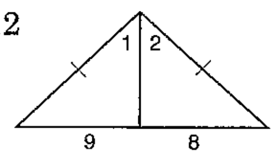
Inequalities Involving Two Triangles

The following two theorems are useful in determining relationships between sides and angles in triangles.

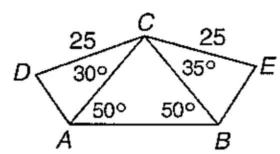
SAS Inequality (Hinge Theorem)	If two sides of one triangle are congruent to two sides of another triangle, and the included angle in one triangle is greater than the included angle in the other, then the third side of the first triangle is longer than the third side in the second triangle.
SSS Inequality	If two sides of one triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.

Examples: Refer to each figure to write an inequality relating the given pair of angle or segment measures.

1 $m\angle 1, m\angle 2$



2 AD, BE

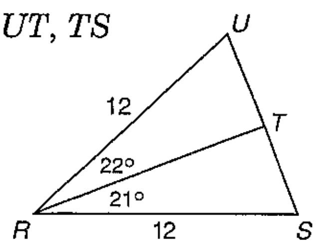


By SSS, $m\angle 1 > m\angle 2$.

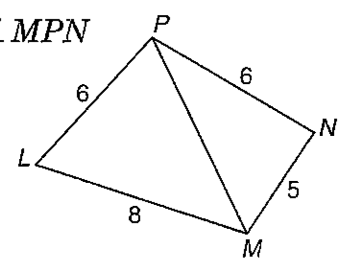
By SAS, $AD < BE$.

Refer to each figure to write an inequality relating the given pair of angle or segment measures.

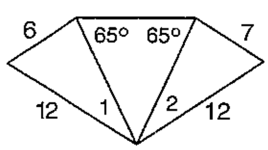
1. UT, TS



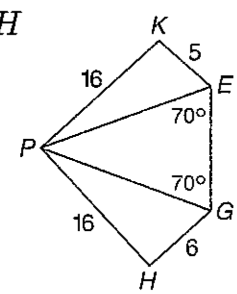
2. $m\angle LPM, m\angle MPN$



3. $m\angle 1, m\angle 2$

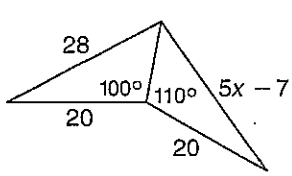


4. $m\angle KPE, m\angle GPH$



Write an inequality or pair of inequalities to describe the possible values of x.

5.



6.