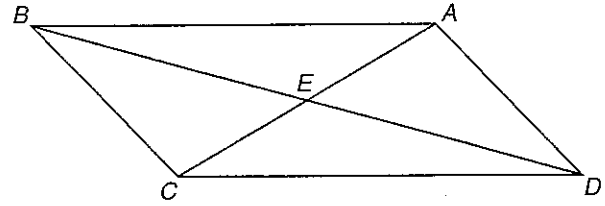


Practice

Parallelograms

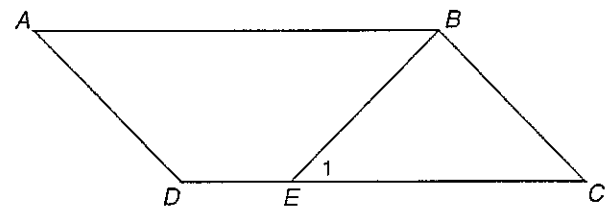
The figure at the right is a parallelogram. Use this figure and the information given to solve each problem.

- If $m\angle BCD = 125$, find $m\angle BAD$.
- If $m\angle BAC = 45$, find $m\angle ACD$.
- If $m\angle BEA = 135$, find $m\angle AED$.
- If $m\angle ABC = 50$, find $m\angle BCD$.
- If $AB = 5x - 3$ and $CD = 2x + 9$, find AB .
- If $m\angle DAB = 2x - 10$ and $m\angle ADC = 3x$, find $m\angle DAB$.
- If $m\angle BAD = 3x - 12$ and $m\angle BCD = x + 40$, find $m\angle BAD$.



- Write a two-column proof.
Given: $ABCD$ is a parallelogram,
 $\overline{BE} \cong \overline{AD}$
Prove: $\angle 1 \cong \angle C$

Proof:



Statements

Reasons

Study Guide

Tests for Parallelograms

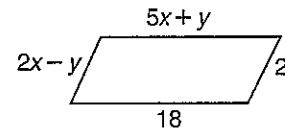
You can show that a quadrilateral is a parallelogram if you can show that one of the following is true.

- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are congruent.
- Diagonals bisect each other.
- Both pairs of opposite angles are congruent.
- A pair of opposite sides are both parallel and congruent.

Example: Find the values of x and y that ensure the quadrilateral is a parallelogram.

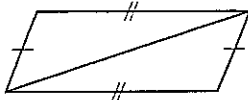
Since opposite sides of a parallelogram must be congruent, then $5x + y = 18$ and $5x - y = 2$.

Solving the system of two equations, you get $x = 2$ and $y = 8$.

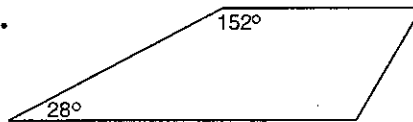


Determine if each quadrilateral is a parallelogram. Justify your answer.

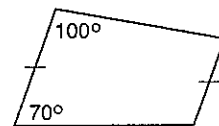
1.



2.

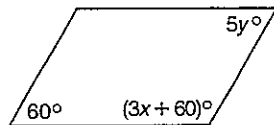


3.

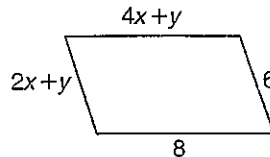


Find the values of x and y that ensure each quadrilateral is a parallelogram.

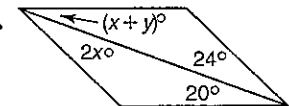
4.



5.



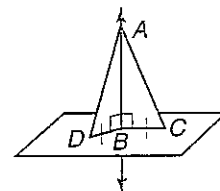
6.



7. Identify the subgoals you would need to accomplish to complete the proof.

Given: $\overline{AB} \perp$ plane BCD .
 $\overline{DB} \cong \overline{CB}$

Prove: $\angle DAB \cong \angle CAB$

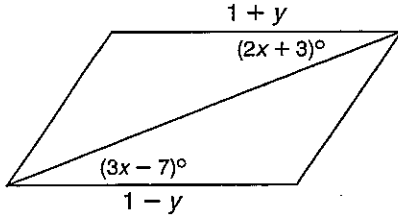


Practice

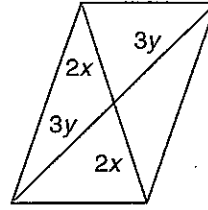
Tests for Parallelograms

Find the values of x and y that insure each quadrilateral is a parallelogram.

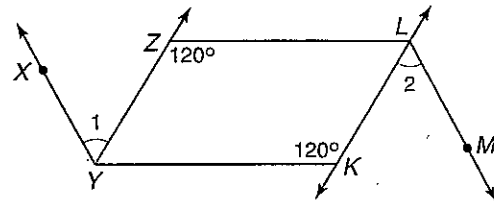
1.



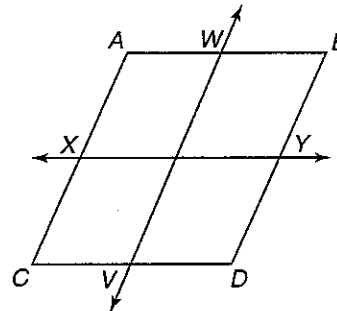
2.



3. Refer to the figure at the right. \overline{YZ} bisects $\angle XYK$ and \overline{LK} bisects $\angle ZLM$. Also, $\angle 1 \cong \angle 2$. Is $YZLK$ a parallelogram? Explain.



4. Refer to the figure at the right. $\overline{AC} \cong \overline{WV}$ and $\overline{BD} \cong \overline{WV}$. Also, $\overline{AB} \cong \overline{XY}$ and $\overline{CD} \cong \overline{XY}$. Is $ABDC$ a parallelogram? Explain.



Determine whether quadrilateral $ABCD$ with the given vertices is a parallelogram. Explain.

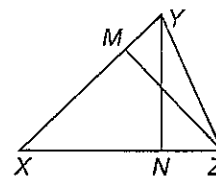
5. $A(2, 5), B(5, 9), C(6, 3), D(3, -1)$

6. $A(-1, 6), B(2, -3), C(5, 9), D(2, 7)$

7. **Identify Subgoals** Identify the subgoals you would need to accomplish to complete the proof.

Given: $\overline{YN} \perp \overline{XZ}, \overline{ZM} \perp \overline{XY}$
 $\overline{XZ} \cong \overline{XY}$
 $\overline{XM} \cong \overline{XN}$

Prove: $\angle XMZ \cong \angle XNY$



Study Guide

Rectangles

A **rectangle** is a quadrilateral with four right angles. Since both pairs of opposite angles are congruent, a rectangle is a parallelogram and has all the properties of a parallelogram. The following list summarizes the properties of a rectangle.

- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.
- All four angles are right angles.
- Diagonals are congruent.

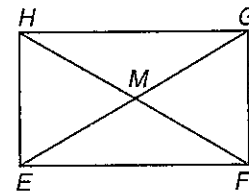
Example: Quadrilateral $EFGH$ is a rectangle. If $EM = 5x + 1$ and $HF = 42$, find the value of x .

Since the diagonals of a rectangle bisect each other and are congruent, you know that $5x + 1 = \frac{1}{2}(42)$.

$$5x + 1 = 21$$

$$5x = 20$$

$$x = 4$$



Quadrilateral $RSTV$ is a rectangle. Find the values of x and y .

1. $VW = 2x + y$

$WS = 36$

$RS = x - y$

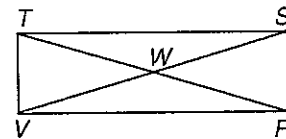
$VT = 9$

2. $VR = y$

$TS = x + 11$

$VT = y - 3x$

$RS = x + 2$



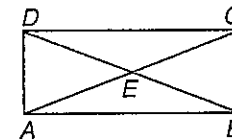
Quadrilateral $ABCD$ is a rectangle. Find the value of x .

3. $m\angle DAC = 4x + 8$

$m\angle CAB = 5x - 8$

4. $AC = x^2$

$DB = 6x - 8$



Determine whether $ABCD$ is a rectangle. Justify your answer.

5. $A(10, 4), B(10, 8),$
 $C(-4, 8), D(-4, 4)$

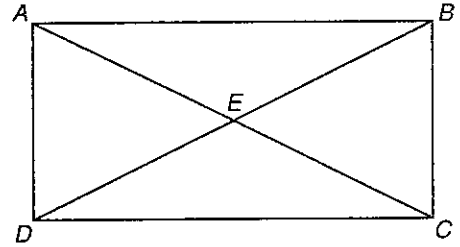
6. $A(3, 7), B(10, 7),$
 $C(11, 12), D(4, 12)$

Practice

Rectangles

Use rectangle $ABCD$ and the given information to solve each problem.

1. If $AC = 4x - 60$ and $BD = 30 - x$, find BD .
2. If $AC = 4x - 60$ and $AE = x + 5$, find EC .
3. If $m\angle BAC = 4x + 5$ and $m\angle CAD = 5x - 14$, find $m\angle CAD$.
4. If $AE = 2x + 3$ and $BE = 12 - x$, find BD .
5. If $m\angle BAC = 3x + 5$ and $m\angle ACD = 40 - 2x$. Find $m\angle AED$.



Determine whether $PQRS$ is a rectangle. Justify your answer.

6. $P(2, 3)$, $Q(5, 9)$, $R(11, 6)$, $S(8, 0)$
7. $P(-1, 4)$, $Q(3, 6)$, $R(9, -3)$, $S(5, -5)$
8. $P(1, 3)$, $Q(4, 7)$, $R(6, 2)$, $S(2, 4)$
9. $P(-1, -3)$, $Q(-4, 6)$, $R(8, 10)$, $S(11, 1)$
10. $P(-1, -2)$, $Q(5, 2)$, $R(13, -10)$, $S(7, -14)$

Study Guide

Squares and Rhombi

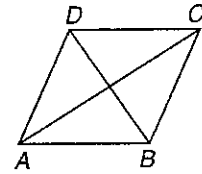
A **rhombus** is a quadrilateral with four congruent sides. A **square** is a quadrilateral with four right angles and four congruent sides.

The diagonals of a rhombus have two special relationships.

- The diagonals of a rhombus are perpendicular.
- Each diagonal of a rhombus bisects a pair of opposite angles.

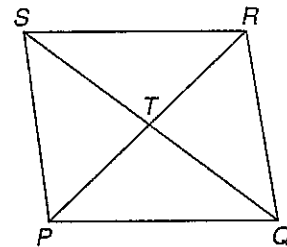
Example: $ABCD$ is a rhombus. If $m\angle ADB = 27$, find $m\angle ADC$.

Since each diagonal of a rhombus bisects a pair of opposite angles, $m\angle ADC = 2(m\angle ADB)$.
So $m\angle ADC = 2(27)$ or 54 .



Use rhombus $PQRS$ and the given information to find each value.

1. If $ST = 13$, find SQ .
2. If $m\angle PRS = 17$, find $m\angle QRS$.
3. Find $m\angle STR$.
4. If $SP = 4x - 3$ and $PQ = 18 + x$, find the value of x .



Determine whether each quadrilateral with the given vertices is a parallelogram, a rectangle, a rhombus, or a square. List all that apply.

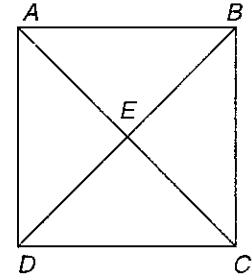
5. $M(1, 5)$, $N(6, 5)$,
 $O(6, 10)$, $P(1, 10)$
6. $W(-4, -2)$, $X(5, -2)$,
 $Y(8, 4)$, $Z(-1, 4)$
7. $D(-7, 3)$, $E(-2, 3)$,
 $F(1, 7)$, $G(-4, 7)$
8. $R(0, 0)$, $E(10, 0)$,
 $S(10, 5)$, $T(0, 5)$

Practice

Squares and Rhombi

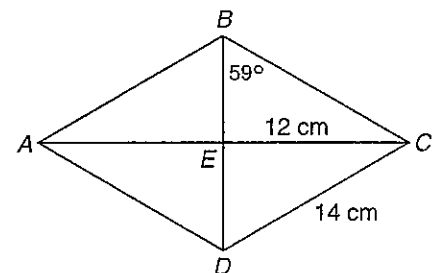
Use square $ABCD$ and the given information to find each value.

1. If $m\angle AEB = 3x$, find x .
2. If $m\angle BAC = 9x$, find x .
3. If $AB = 2x + 4$ and $CD = 3x - 5$, find BC .
4. If $m\angle DAC = y$ and $m\angle BAC = 3x$, find x and y .
5. If $AB = x^2 - 15$ and $BC = 2x$, find x .



Use rhombus $ABCD$ and the given information to find each measure.

6. $m\angle BCE$
7. $m\angle BEC$
8. AC
9. $m\angle ABD$
10. AD



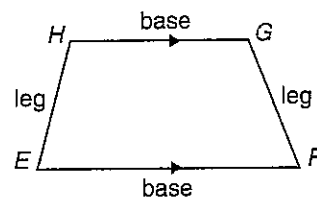
Determine whether $EFGH$ is a parallelogram, a rectangle, a rhombus, or a square for each set of vertices. List all that apply.

11. $E(0, -3), F(-3, 0), G(0, 3), H(3, 0)$
12. $E(2, 1), F(3, 4), G(7, 2), H(6, -1)$

Study Guide

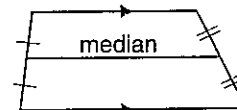
Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**, and the nonparallel sides are called **legs**. In trapezoid $EFGH$, $\angle E$ and $\angle F$ are called **base angles**. $\angle H$ and $\angle G$ form the other pair of base angles.



A trapezoid is an **isosceles trapezoid** if its legs are congruent.

The **median** of a trapezoid is the segment that joins the midpoints of the legs.

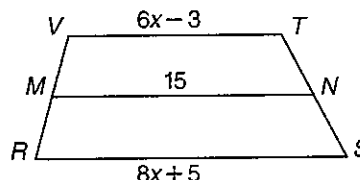


The following theorems about trapezoids can be proved.

- Both pairs of base angles of an isosceles trapezoid are congruent.
- The diagonals of an isosceles trapezoid are congruent.
- The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

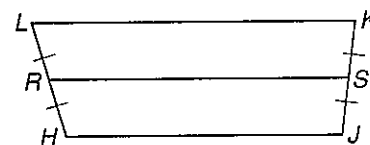
Example: Given trapezoid $RSTV$ with median \overline{MN} , find the value of x .

$$\begin{aligned} MN &= \frac{1}{2}(VT + RS) \\ 15 &= \frac{1}{2}(6x - 3 + 8x + 5) \\ 15 &= \frac{1}{2}(14x + 2) \\ 15 &= 7x + 1 \\ 14 &= 7x \\ 2 &= x \end{aligned}$$



$HJKL$ is an isosceles trapezoid with bases \overline{HJ} and \overline{LK} , and median \overline{RS} . Use the given information to solve each problem.

1. If $LK = 30$ and $HJ = 42$, find RS .
2. If $RS = 17$ and $HJ = 14$, find LK .
3. If $RS = x + 5$ and $HJ + LK = 4x + 6$, find RS .
4. If $m\angle LRS = 66$, find $m\angle KSR$.



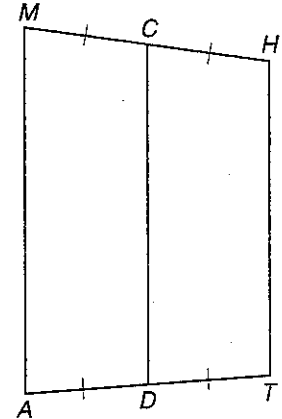
5. Find the length of the median of a trapezoid with vertices at $C(3, 1)$, $D(10, 1)$, $E(7, 9)$, and $F(5, 9)$.

Practice

Trapezoids

MATH is an isosceles trapezoid with bases \overline{MA} and \overline{TH} . Use the given information to solve each problem.

- If $MA = 34$ and $HT = 20$, find CD .
- If $HT = 17.6$ and $CD = 28.6$ find MA .
- If $MA = 23.9$ and $CD = 16.4$, find HT .
- If $CD = x + 12$ and $MA + HT = 4x + 3$, find x .
- If $m\angle TAM = 63$, find $m\angle HMA$.
- If $m\angle HCD = 52$, find $m\angle TDC$.
- If $m\angle DCM = 2x$, find $m\angle CMA$ in terms of x .



- If the measure of the median of an isosceles trapezoid is 5.5, what are the possible integral measures for the bases?

- \overline{VW} is the median of a trapezoid that has bases \overline{MN} and \overline{PO} , with V on \overline{OM} and W on \overline{PN} . If the vertices of the trapezoid are $M(2, 6)$, $N(4, 6)$, $P(10, 0)$, and $O(0, 0)$, find the coordinates of V and W .
- \overline{VW} is the median of a trapezoid that has bases \overline{MN} and \overline{PO} , with V on \overline{PM} and W on \overline{ON} . If four of the points are $M(5, 10)$, $N(9, 10)$, $V(3, 7)$, and $W(11, 7)$, find the coordinates of P and O .