

Study Guide

Integration: Algebra
Using Proportions

A **ratio** is a comparison of two quantities. The ratio of a to b can be expressed as $\frac{a}{b}$, where b is not 0. The ratio can also be written $a:b$.

An equation stating that two ratios are equal is a **proportion**. Therefore, $\frac{a}{b} = \frac{c}{d}$ is a proportion for any numbers a and c and any nonzero numbers b and d . In any true proportion, the cross products are equal. So, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Example: Solve $\frac{11}{16} = \frac{44}{x}$ by using cross products.

$$\begin{aligned}\frac{11}{16} &= \frac{44}{x} \\ 11x &= 16 \cdot 44 \\ 11x &= 704 \\ x &= 64\end{aligned}$$

For Exercises 1–4, use the table to find the ratios. Express each ratio as a decimal rounded to three places.

Teams	Wins	Losses
Hawks	16	13
Tigers	15	14
Mustangs	12	16

- games won to games lost for Hawks
- games won by the Hawks to games won by Tigers
- games won to games played for Tigers
- games won to games played for Mustangs

Solve each proportion by using cross products.

$$5. \frac{9}{28} = \frac{x}{84}$$

$$6. \frac{3}{18} = \frac{4x}{7}$$

$$7. \frac{x+5}{7} = \frac{x+3}{5}$$

Use a proportion to solve each problem.

- If two cassettes cost \$14.50, how much will 15 cassettes cost?
- If a 6-foot post casts a shadow that is 8 feet long, how tall is an antenna that casts a 60-foot shadow at the same time?

Practice

**Integration: Algebra
Using Proportions***Solve each proportion using cross products.*

1. $\frac{3}{5} = \frac{x}{15}$

2. $\frac{20-x}{x} = \frac{6}{4}$

3. $\frac{x+1}{5} = \frac{x-1}{2}$

4. $\frac{x}{x-3} = \frac{x+4}{x}$

5. $\frac{x+1}{6} = \frac{x-1}{x}$

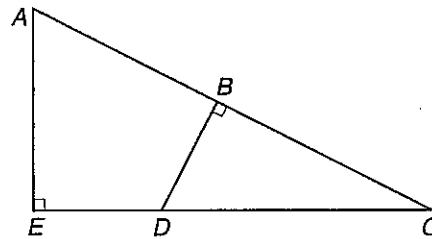
6. $\frac{1}{x} = \frac{6}{x+9}$

7. $\frac{x}{x+8} = \frac{2}{3}$

8. $\frac{4}{12} = \frac{x+2}{2x+5}$

In the figure at the right, $\frac{AC}{CD} = \frac{CE}{CB}$. Use proportions to complete the table.

	AC	BC	AB	CE	ED	DC
9.	10	4		8		
10.	12			10		9

*Use a proportion to solve each problem.*

- The ratio of seniors to juniors in the Math Club is 2:3. If there are 21 juniors, how many seniors are in the club?
- A 15-foot building casts a 9-foot shadow. How tall is a building that casts a 30-foot shadow at the same time?
- A photo that is 3 inches wide and 5 inches high was enlarged so that it is 12 inches wide. How high is the enlargement?
- Philip has been eating 2 hamburgers every 5 days. At that rate, how many hamburgers will he eat in 30 days?

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Exploring Similar Polygons

Two polygons are **similar** if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

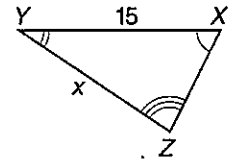
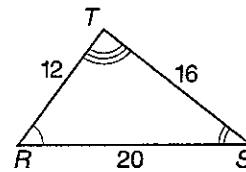
The symbol \sim means *is similar to*.

The ratio of the lengths of two corresponding sides of two similar polygons is called the **scale factor**.

Example: Find x if $\triangle RST \sim \triangle XYZ$.

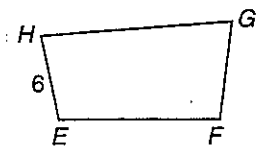
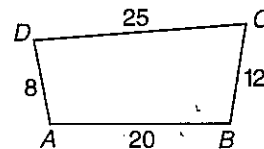
The corresponding sides are proportional, so we can write a proportion to find the value of x .

$$\begin{aligned}\frac{16}{x} &= \frac{20}{15} \\ 20x &= 240 \\ x &= 12\end{aligned}$$



If quadrilateral $ABCD$ is similar to quadrilateral $EFGH$, find each of the following.

1. scale factor of $ABCD$ to $EFGH$



2. EF

3. FG

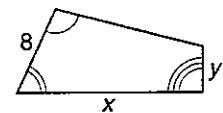
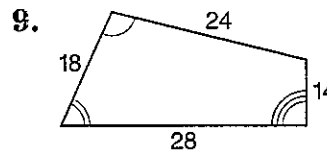
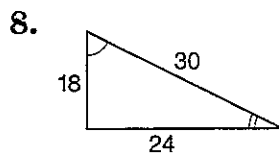
4. GH

5. perimeter of $ABCD$

6. perimeter of $EFGH$

7. ratio of perimeter of $ABCD$ to perimeter of $EFGH$

Each pair of polygons is similar. Find the values of x and y .

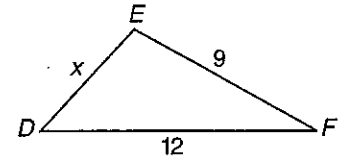


Practice

Exploring Similar Polygons

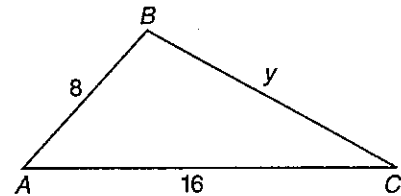
In the figure at the right, $\triangle ABC$ is similar to $\triangle DEF$.

1. Write three equal ratios to show corresponding sides are proportional.



2. Find the value of x .

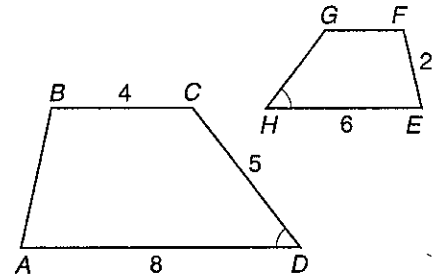
3. Find the value of y .



4. Find the ratio $\frac{m\angle A}{m\angle D}$.

In the figure at the right, quadrilateral $ABCD$ is similar to quadrilateral $EFGH$.

5. Write four equal ratios to show corresponding sides are proportional.



6. Find AB .

7. Find HG .

8. Find FG .

9. The sum of the measures of $\angle A$ and $\angle C$ equals the sum of the measures of which two angles of quadrilateral $EFGH$?

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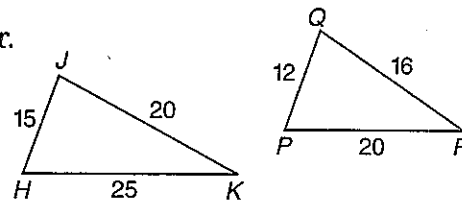
Identifying Similar Triangles

There are three ways to determine whether two triangles are similar.

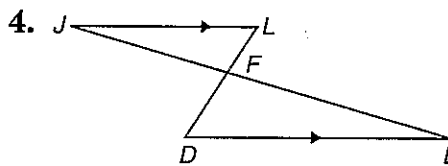
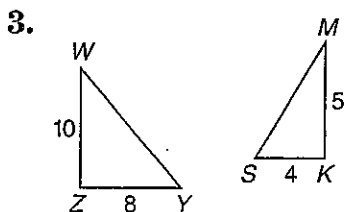
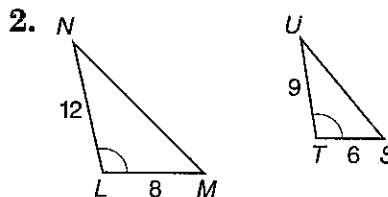
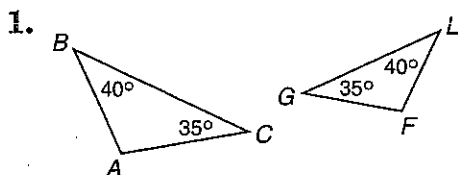
- Show that two angles of one triangle are congruent to two angles of the other triangle. (AA Similarity)
- Show that the measures of the corresponding sides of the triangles are proportional. (SSS Similarity)
- Show that the measure of two sides of a triangle are proportional to the measures of the corresponding sides of the other triangle and that the included angles are congruent. (SAS Similarity)

Example: Determine whether the triangles are similar. Explain your answer.

Since $\frac{15}{12} = \frac{25}{20} = \frac{20}{16}$, the triangles are similar by SSS Similarity.

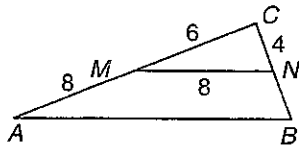


Determine whether each pair of triangles is similar. Give a reason for your answer.

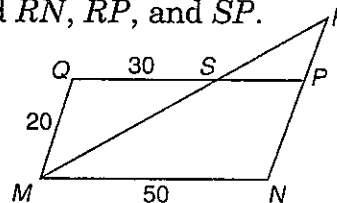


Identify the similar triangles in each figure. Explain why they are similar and find the missing measures.

5. If $\overline{MN} \parallel \overline{AB}$, find AB , BC , and BN .



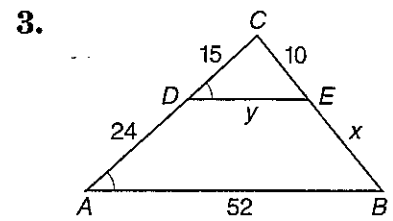
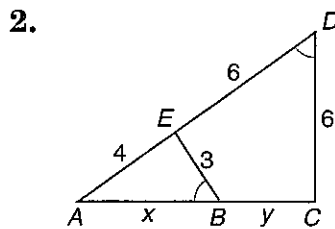
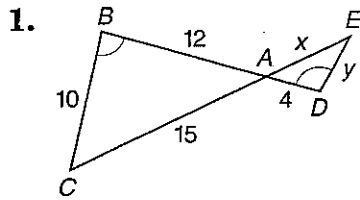
6. If $MNPQ$ is a parallelogram, find RN , RP , and SP .



Practice

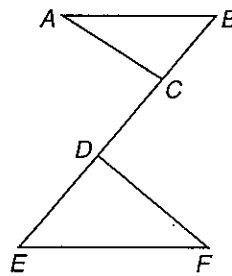
Identifying Similar Triangles

Identify the similar triangles in each figure. Explain why they are similar and use the given information to find x and y .



Write a two-column proof.

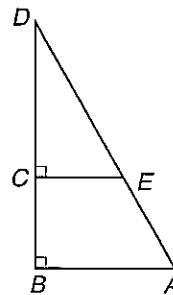
4. Given: $\overline{AB} \parallel \overline{EF}$
 $\overline{AC} \parallel \overline{DF}$
 Prove: $\triangle ABC \sim \triangle FED$



Proof:
Statements

Reasons

5. Given: $\overline{AB} \perp \overline{BD}$
 $\overline{EC} \perp \overline{BD}$
 Prove: $\triangle BDA \sim \triangle CDE$



Proof:
Statements

Reasons

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Parallel Lines and Proportional Parts

The following theorems involve proportional parts of triangles.

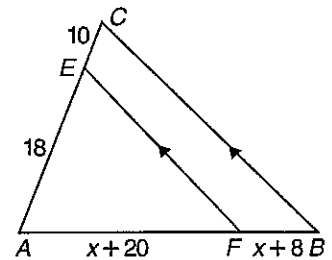
- If a line is parallel to one side of a triangle and intersects the other two sides, then it separates these sides into segments of proportional lengths.
- If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.
- A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle and its length is one-half the length of the third side.

Example: In $\triangle ABC$, $\overline{EF} \parallel \overline{CB}$, find the value of x .

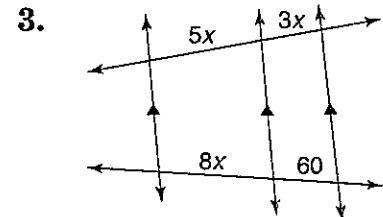
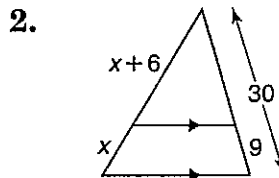
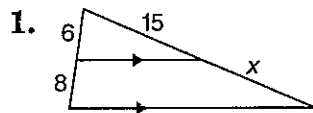
$$\overline{EF} \parallel \overline{CB} \text{ implies that } \frac{AF}{FB} = \frac{AE}{EC}.$$

Rewrite the proportion and solve.

$$\begin{aligned} \frac{x+20}{x+8} &= \frac{18}{10} \\ 10x + 200 &= 18x + 144 \\ 56 &= 8x \\ 7 &= x \end{aligned}$$



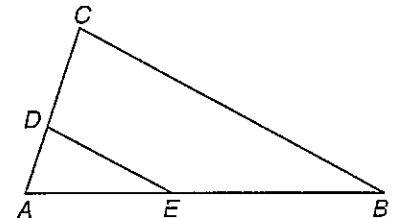
Find the value of x .



In $\triangle ABC$, find x so that $\overline{DE} \parallel \overline{CB}$.

4. $DC = 18$, $AD = 6$,
 $AE = 12$, $EB = x - 3$

5. $AC = 30$, $AD = 10$,
 $AE = 22$, $EB = x + 4$



6. In $\triangle RST$, M is the midpoint of \overline{RS} , N is the midpoint of \overline{ST} , and P is the midpoint of \overline{RT} . Find the perimeter of $\triangle MNP$ if $RS = 28$, $ST = 34$, and $RT = 30$.

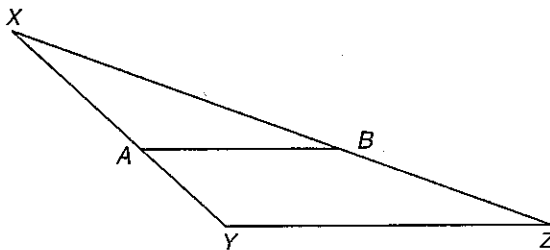
Practice

Parallel Lines and Proportional Parts

Refer to the figure at the right for Exercises 1-2. Determine whether it is always true that $\overline{AB} \parallel \overline{YZ}$ under the given conditions.

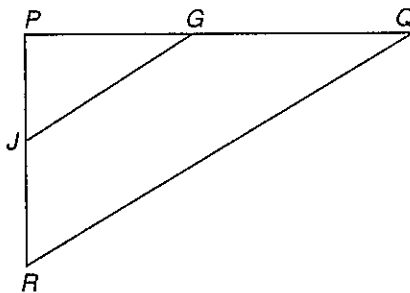
1. $XA = 6$
 $AY = 4$
 $XB = 8$
 $BZ = 5$

2. $XB = 3$
 $BZ = 2$
 $AX = 6$
 $AY = 10$



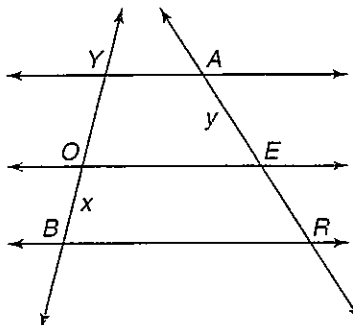
In $\triangle PQR$, find x and y so that $\overline{JG} \parallel \overline{RQ}$.

3. $PJ = 6$
 $JG = 5$
 $PG = 4$
 $GQ = x$
 $RQ = x + 6$
 $JR = y$



4. $RQ = 10$
 $JG = 8$
 $PJ = 8x - 5$
 $JR = x$
 $PG = 3y + 2$
 $QG = y$

5. In the figure at the right, $\overline{YA} \parallel \overline{OE} \parallel \overline{BR}$. Find the values of x and y if $YO = 4$, $ER = 16$, and $AR = 24$.



Study Guide

Parts of Similar Triangles

Each mathematical word in the following list has a different meaning in everyday usage than when it is used in a mathematical context. When you read mathematics, you must be sure that you are using each word in the correct context.

altitude	right	mean	scale
extreme	product	segment	ruler
plane	obtuse	acute	
meter	yard	median	

Each definition below describes an everyday usage of one of the words in the list above. For each definition, write the correct word from the list above in the blanks at the right.

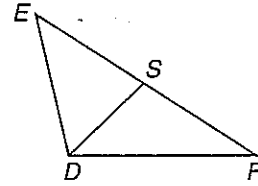
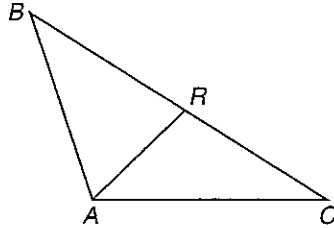
1. an instrument for measuring _____
2. a tool for smoothing a wood surface _____
3. the opposite of left _____
4. differing widely from the ordinary _____
5. keen in perception _____
6. dull _____
7. contemptible _____
8. fragment _____
9. the dividing strip down the middle of a highway _____
10. height above sea level _____
11. a person in charge of a country _____
12. the grounds of a building _____
13. an item that is manufactured _____
14. a small plate forming part of the external covering of a fish _____
15. Choose five words from the list above. Compare their mathematical definition and their everyday usage. Describe how the definitions are similar and how they are different.

Practice

Parts of Similar Triangles

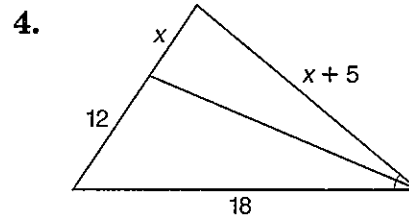
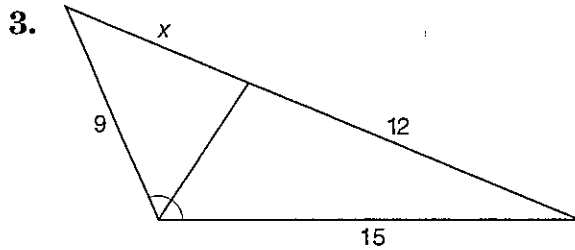
In the figure at the right, $\triangle ABC \sim \triangle DEF$, $\overline{BR} \cong \overline{RC}$, and $\overline{ES} \cong \overline{SF}$. Find the value of x .

1. $BC = 24$
 $EF = 15$
 $AR = x$
 $DS = x - 6$



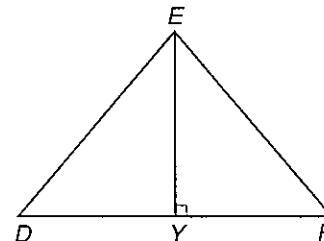
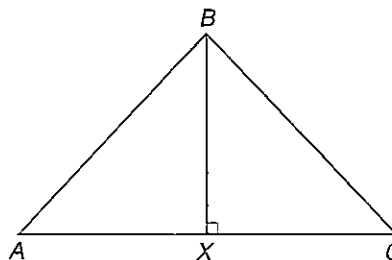
2. $AB = 2x + 5$
 $DE = x + 7$
 $AR = 24$
 $DS = 18$

Find the value of x .



In the figure at the right, $\triangle ABC \sim \triangle DEF$, and \overline{BX} and \overline{EY} are altitudes. Find the value of x .

5. $AB = 25$
 $DE = 16$
 $BX = 18$
 $EY = x$



6. $AB = 30$
 $DE = 25$
 $BX = 2x + 5$
 $EY = x + 10$

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Pages 378–383

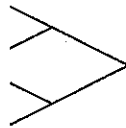
Fractals and Self-Similarity

Fractal geometry is the geometry of things in nature that are irregular in shape. A **fractal** is a geometric shape created using a process called **iteration**. Iteration is a process of repeating the same procedure over and over again. **Self-similarity** is a characteristic of fractals. The smaller and smaller details of a shape have the same geometrical characteristics as the original, larger form.

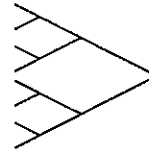
Example:



Stage 1



Stage 2

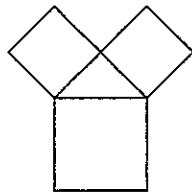


Stage 3

1. Follow the iteration process to produce a figure.

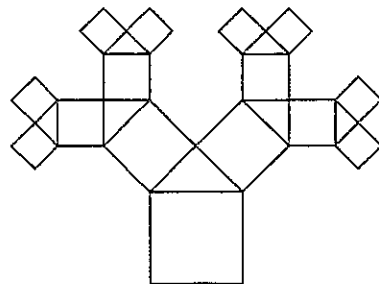
- Draw a square.
- Attach the hypotenuse of an isosceles right triangle to one side of the square. The hypotenuse should be the same length as the side of the square.
- Attach a square to each leg of the triangle. The sides of the squares should be the same length as the legs of the triangle.

This is Stage 1.



2. Describe the next step in the iteration process.

3. Draw Stage 3 of the figure in Exercise 1.



4. Is the figure produced in Exercise 3 self-similar?

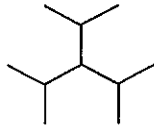
Practice

Fractals and Self-Similarity

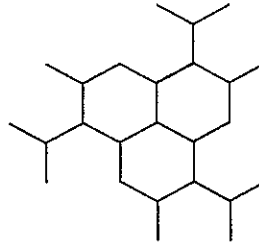
Use the drawings below of three stages of a variation on the Koch curve to complete Exercises 1 and 2.



Stage 1



Stage 2



Stage 3

1. Draw Stage 4.
2. Describe the iterative process used in this variation.
3. Draw an equilateral triangle. Divide each side into fourths, and connect the points to form three lines parallel to each side of the triangle.
4. Find the number of similar triangles within the figure in Exercise 3.
5. Create a figure and draw stages 1-3 of iteration.
6. Describe the iterative process used in Exercise 5.
7. **Solve a Simpler Problem** How many diagonals can be drawn for a polygon with 15 sides?