NAME ______________________________ DATE ____________

8–1

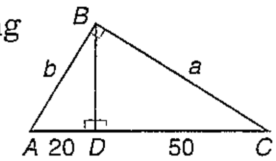
Study Guide

Student Edition
Pages 397–404

Geometric Mean and the Pythagorean Theorem

The geometric mean between two positive numbers a and b is the positive number x where $\frac{a}{x} = \frac{x}{b}$.

If $\triangle ABC$ is a right triangle with altitude $\overline{BD}$, then the following relationships hold true.



$$\frac{AD}{BD} = \frac{BD}{DC} \qquad \frac{AC}{BC} = \frac{BC}{DC} \qquad \frac{AC}{AB} = \frac{AB}{AD}$$

You can use the Pythagorean Theorem to find missing measures for right triangles.

Pythagorean Theorem	In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
Converse of the Pythagorean Theorem	If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Use $\triangle ABC$ above for the following examples.

Examples: **1** Find a.

You can use geometric mean relationships.

$$\frac{AC}{BC} = \frac{BC}{DC}$$
$$\frac{70}{a} = \frac{a}{50}$$
$$a^2 = 3500$$
$$a = \sqrt{3500}$$
$$a \approx 59.2$$

2 Find b.

You can use the Pythagorean Theorem.

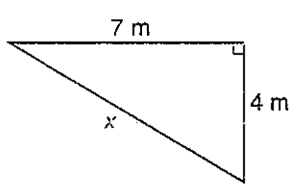
$$b^2 + a^2 = 70^2$$
$$b^2 + 3500 = 4900$$
$$b^2 = 1400$$
$$b = \sqrt{1400}$$
$$b \approx 37.4$$

Find the geometric mean between each pair of numbers.

1. 3 and 10 **2.** 10 and 20 **3.** 10 and 40

Find the values of x and y. Round to the nearest tenth.

4.



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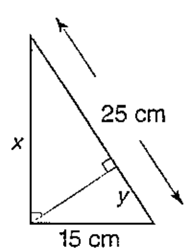

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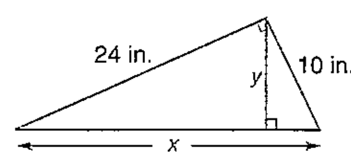

Determine if the given measures are measures of the sides of a right triangle.

7. 18, 24, 30 **8.** 20, 30, 40 **9.** 4.5, 6, 7.5

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NAME ______________________________ DATE ______________

Practice

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Pages 397–404

Geometric Mean and the Pythagorean Theorem

Find the geometric mean between each pair of numbers.

1. 5 and 10

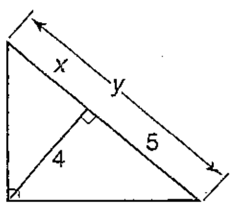
2. 3 and 27

3. 6 and $\frac{1}{2}$

4. 16 and $\frac{1}{9}$

Find the values of x and y.

5.



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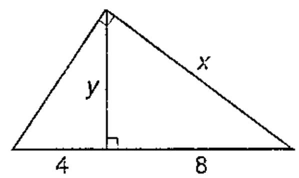

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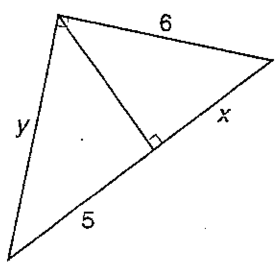

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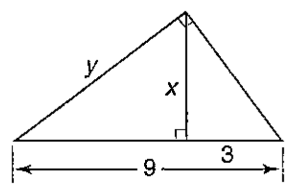

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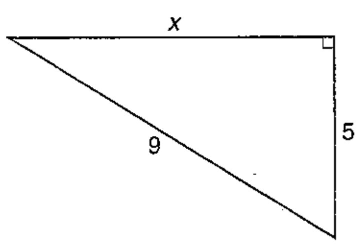

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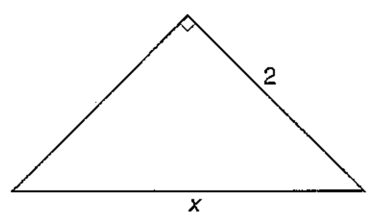

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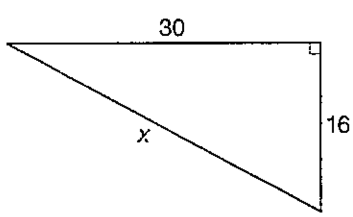

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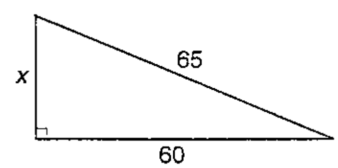


Determine if the given measures are measures of the sides of a right triangle.

13. 14, 48, 50

14. 50, 75, 85

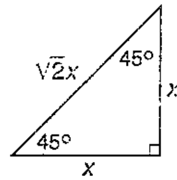
15. 15, 36, 39

16. 45, 60, 80

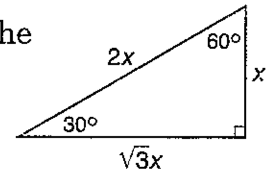
Study Guide

Special Right Triangles

Two special kinds of right triangles are the 45°-45°-90° triangle and the 30°-60°-90° right triangle.

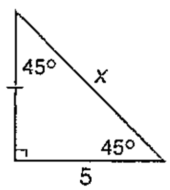


- In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg.
- In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Examples: Find the value of x.

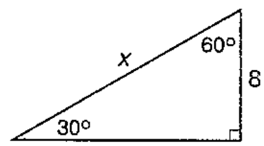
1



Since the triangle is a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as the leg.

So $x = 5\sqrt{2}$ or about 7.1.

2



Since the triangle is a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg.

So $x = 2(8)$ or 16.

Find the value of x.

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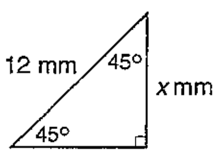

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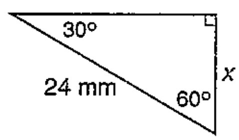

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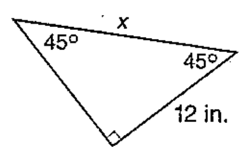

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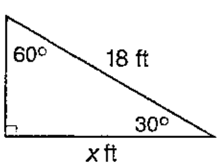

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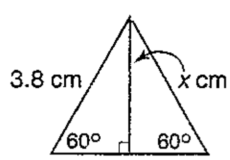

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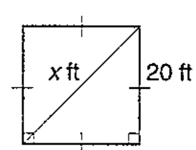
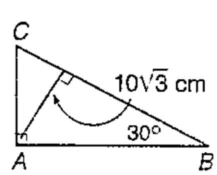


7. Find the perimeter of the triangle shown at the right.



Practice

Special Right Triangles

Find the values of x and y.

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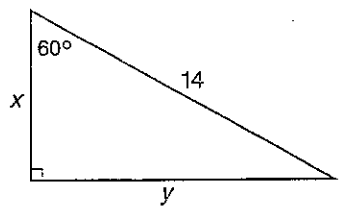

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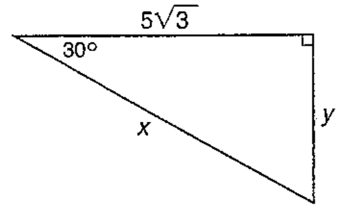

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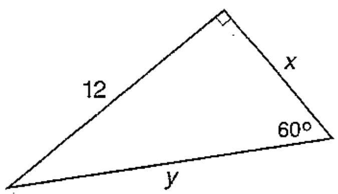

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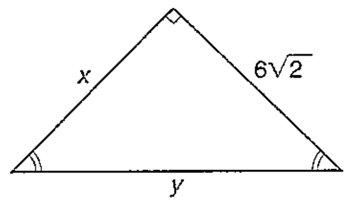

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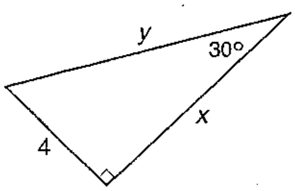

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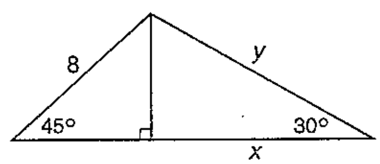

7. Find the length of a diagonal of a square with sides 10 in. long.

8. Find the length of a side of a square whose diagonal is 4 cm.

9. One side of an equilateral triangle measures 6 cm. Find the measure of an altitude of the triangle.

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Study Guide

Student Edition
Pages 412–419

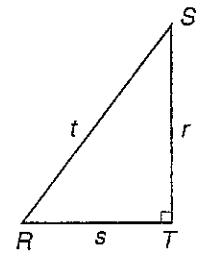
Integration: Trigonometry Ratios in Right Triangles

A ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**. Their abbreviations are *sin*, *cos*, and *tan*, respectively. These ratios are defined for the acute angles of right triangles, though your calculator will give the values of sine, cosine, and tangent for angles of greater measure.

$$\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \frac{r}{t}$$

$$\cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{s}{t}$$

$$\tan R = \frac{\text{leg opposite to } \angle R}{\text{leg adjacent to } \angle R} = \frac{r}{s}$$

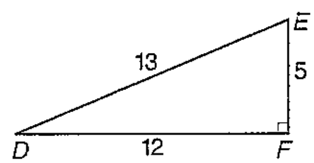


Example: Find sin D, cos D, and tan D. Express each ratio as a fraction and as a decimal rounded to the nearest thousandth.

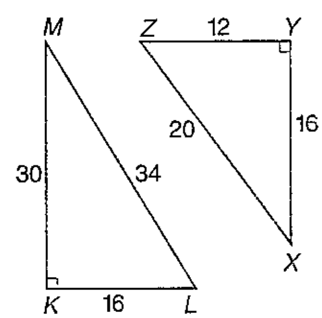
$$\sin D = \frac{5}{13} \approx 0.385$$

$$\cos D = \frac{12}{13} \approx 0.923$$

$$\tan D = \frac{5}{12} \approx 0.417$$



Find the indicated trigonometric ratio as a fraction and as a decimal rounded to the nearest ten-thousandth.



1. sin M
2. cos Z
3. tan L
4. sin X
5. cos L
6. tan Z

Find the value of each ratio to the nearest ten-thousandth.

7. sin 12°
8. cos 32°
9. tan 74°
10. sin 55°

8-3

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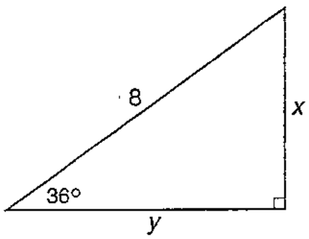
Practice

Student Edition
Pages 412–419

Integration: Trigonometry Ratios in Right Triangles

Find the values of x and y. Round to the nearest tenth.

1.



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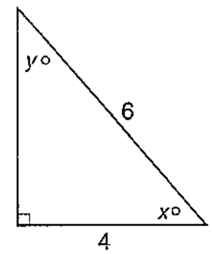

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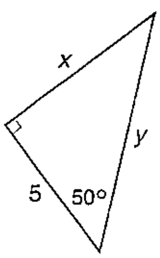

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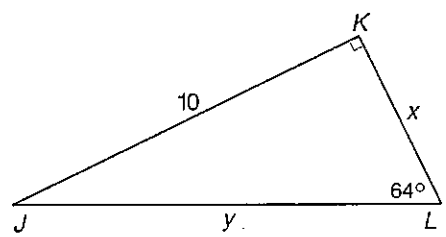

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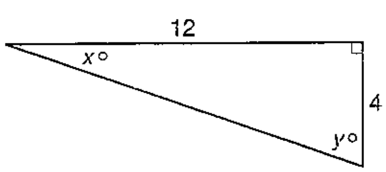

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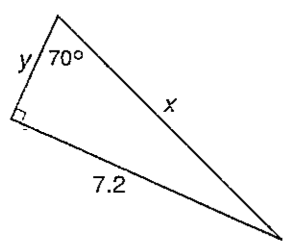

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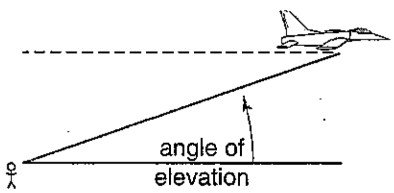
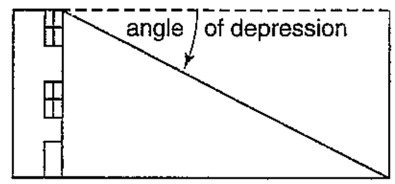
Study Guide

Student Edition
Pages 420–425

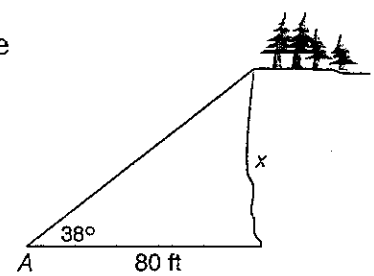
Angles of Elevation and Depression

Many problems in daily life can be solved by using trigonometry. Often such problems involve an **angle of elevation** or an **angle of depression**.





Example: The angle of elevation from point A to the top of a cliff is 38°. If point A is 80 feet from the base of the cliff, how high is the cliff?



Let x represent the height of the cliff.
Then $\tan 38° = \frac{x}{80}$.

$80 \tan 38° = x$

Use a calculator set for the degree mode to find x.

ENTER: 80 ⊠ 38 TAN ⊟ 62.502850

The cliff is about 63 feet high.

Solve each problem. Round measures of segments to the nearest hundredth and measures of angles to the nearest degree.

1. From the top of a tower, the angle of depression to a stake on the ground is 72°. The top of the tower is 80 feet above ground. How far is the stake from the foot of the tower?

2. A tree 40 feet high casts a shadow 58 feet long. Find the measure of the angle of elevation of the sun.

3. A ladder leaning against a house makes an angle of 60° with the ground. The foot of the ladder is 7 feet from the foundation of the house. How long is the ladder?

4. A balloon on a 40-foot string makes an angle of 50° with the ground. How high above the ground is the balloon if the hand of the person holding the balloon is 6 feet above the ground?

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8–4

Practice

Student Edition
Pages 420–425

Angles of Elevation and Depression

Solve each problem. Round measures of segments to the nearest hundredth and measures of angles to the nearest degree.

1. A 20-foot ladder leans against a wall so that the base of the ladder is 8 feet from the base of the building. What angle does the ladder make with the ground?

2. A 50-meter vertical tower is braced with a cable secured at the top of the tower and tied 30 meters from the base. What angles does the cable form with the vertical tower?

3. At a point on the ground 50 feet from the foot of a tree, the angle of elevation to the top of the tree is 53°. Find the height of the tree.

4. From the top of a lighthouse 210 feet high, the angle of depression to a boat is 27°. Find the distance from the boat to the food of the lighthouse. The lighthouse was built at sea level.

5. Richard is flying a kite. The kite string makes an angle of 57° with the ground. If Richard is standing 100 feet from the point on the ground directly below the kite, find the length of the kite string.

6. An airplane rises vertically 1000 feet over a horizontal distance of 1 mile. What is the angle of elevation of the airplane's path? (Hint: 1 mile = 5280 feet)

8-5

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Study Guide

Student Edition
Pages 426–430

Using the Law of Sines

Trigonometric functions can also be used to solve problems that involve triangles that are not right triangles.

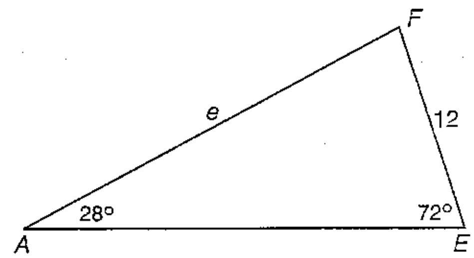
Law of Sines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Example: In $\triangle DEF$ find e.

$$\frac{\sin 28°}{12} = \frac{\sin 72°}{e}$$ **Use the Law of Sines.**

$$e \sin 28° = 12 \sin 72°$$

$$e = \frac{12 \sin 72°}{\sin 28°}$$



Use a calculator to find e.

ENTER: 12 [×] 72 [SIN] [÷] 28 [SIN] [=] 24.30962618

So, $e \approx 24.3$.

Draw △RST and mark it with the given information. Write an equation that could be used to find each unknown value. Then find the value to the nearest tenth.

1. If $s = 18$, $m\angle R = 32$, and $m\angle S = 47$, find r.

2. If $s = 42$, $t = 29$, and $m\angle S = 63$, find $m\angle T$.

3. If $m\angle R = 40$, $m\angle S = 89$, and $t = 4.8$, find r.

4. If $m\angle R = 46$, $m\angle S = 85$, and $t = 17$, find s.

5. Solve $\triangle ABC$ if $a = 15$, $c = 18$, and $m\angle C = 68$. Round measures to the nearest tenth.

Practice

Student Edition
Pages 426–430

Using the Law of Sines

Solve each △ABC. Round measures to the nearest tenth.

1. $a = 12, m\angle B = 70, m\angle C = 15$

2. $a = 12, b = 5, m\angle A = 110$

3. $a = 8, m\angle A = 60, m\angle C = 40$

4. $a = 5, c = 4, m\angle A = 65$

5. $b = 6, m\angle A = 44, m\angle B = 68$

6. $a = 7, m\angle A = 37, m\angle B = 76$

7. $a = 9, b = 9, m\angle C = 20°$

8. A ship is sighted from two radar stations 43 km apart. The angle between the line segment joining the two stations and the radar beam of the first station is 37°. The angle between the line segment joining the two stations and the beam from the second station is 113°. How far is the ship from the second station?

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Study Guide

Using the Law of Cosines

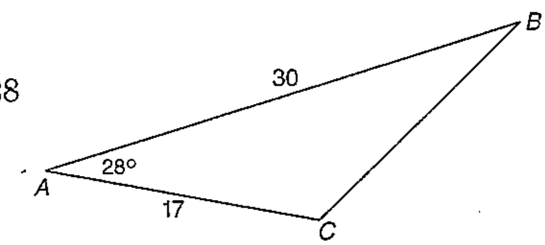
The Law of Cosines often allows you to solve a triangle when the Law of Sines cannot be used.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then, the following equations hold true. $a^2 = b^2 + d^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example: For $\triangle ABC$, find a if $m\angle A = 28$, $c = 30$, and $b = 17$.

Since the measures of two sides and the included angle are known, this is an example of Case 1 for the Law of Cosines.

$a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = 17^2 + 30^2 - 2(17)(30) \cos 28$
$a^2 = 289 + 900 - 1020 \cos 28$
$a^2 \approx 288.393$
$a \approx 16.98$



Use a calculator to solve each triangle ABC described below. Round measures to the nearest tenth.

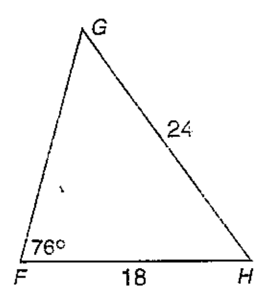
1. $m\angle C = 60$, $a = 12$, $b = 15$

2. $a = 34$, $c = 27$, $m\angle B = 60$

3. $m\angle C = 65$, $a = 8.4$, $b = 9.6$

4. $a = 5$, $b = 9$, $c = 10$

5. Decision Making State whether you would use the Law of Sines or the Law of Cosines to solve $\triangle FGH$.



Practice

Using the Law of Cosines

Solve each triangle △ABC described below. Round measures to the nearest tenth.

1. $a = 16, b = 20, m\angle B = 40$

2. $a = 10, b = 15, c = 12$

3. $a = 42, c = 60, m\angle B = 58$

4. $m\angle A = 60, m\angle B = 72, c = 9$

5. $a = 7, b = 12, c = 15$

6. $m\angle A = 43, b = 23, c = 26$

7. $a = 16, m\angle A = 23, m\angle B = 87$

8. $c = 15.6, a = 12.9, b = 18.4$

9. **Decision-Making** Complete the addition problem at the right. If a letter is used more than once, it represents the same digit each time.

```
  SOME
+MORE
 SENSE
```